Achieving Practical Post-Quantum Threshold Signatures from Lattices

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PhD defense - 19/05/2026

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Cryptography

Science of securing communications in the presence of adversarial behavior.

Digital Signatures: Authenticating Documents

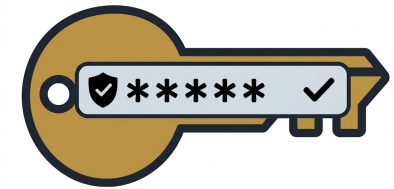
Real-life analogy:

- Only you know how to write your signature.
- Anyone can verify it by comparing against another sample.

1. Introduction

Digital Signatures: Authenticating Documents

Private Key, kept confidential.
Used for signature creation.



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Used for signature verification.

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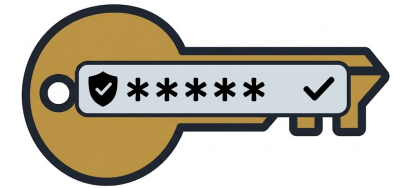

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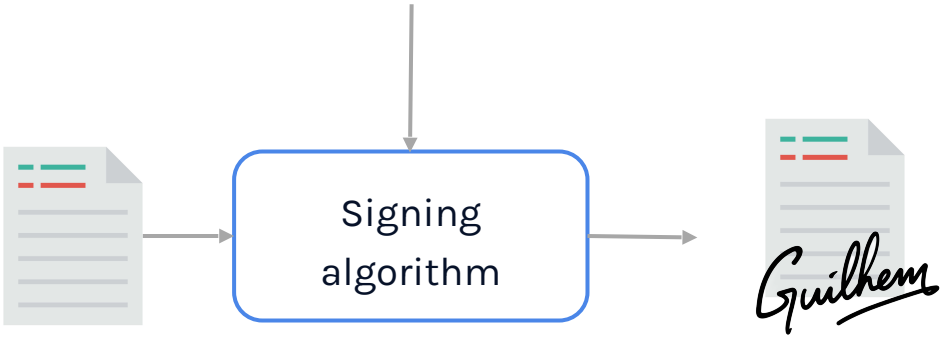

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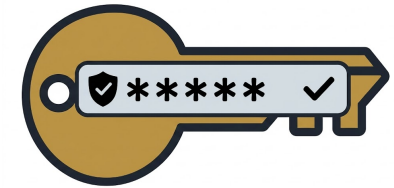

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The Need For Post-Quantum Cryptography

Classical Cryptography

RSA: based on the factorization problem. Given $n = p \cdot q$, hard to find p and q.

Elliptic curves: based on the discrete logarithm problem. Given g^a, hard to find a.

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The Threat of Quantum Computing

Classical bit

Quantum bit (qubit)

Entanglement: Correlate the probabilities of several qubits.

Allows parallel processing and exponential speedups for some problems.

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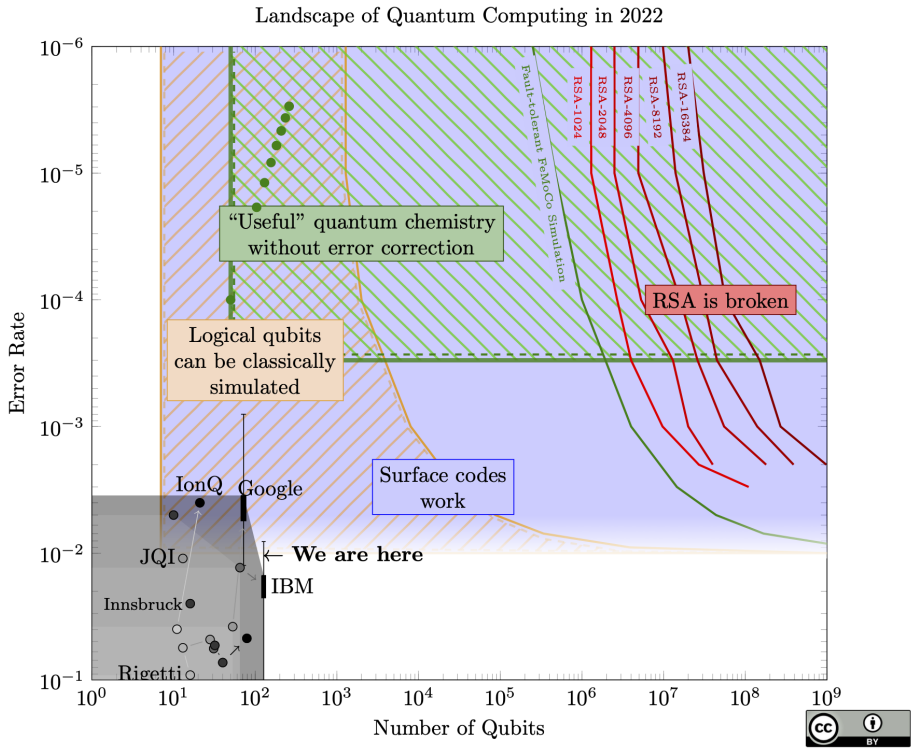
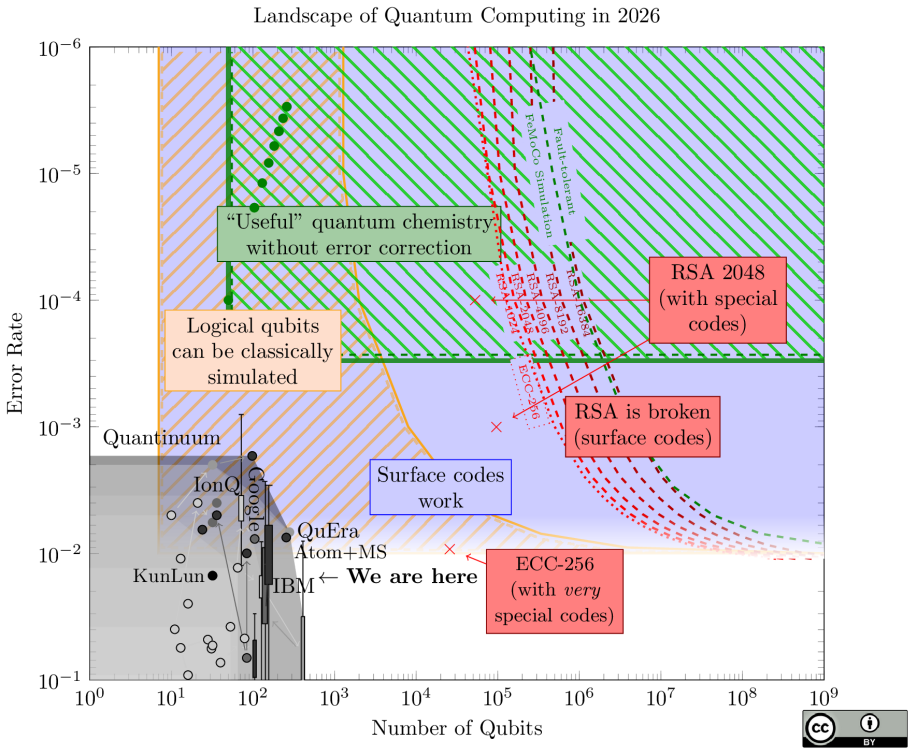
Shor's algorithm (1994)

Efficiently breaks factorization and discrete logarithm problems with qubits…





State of Quantum Computing, by *Samuel Jaques*

1. Introduction

NIST Standardization Process

2017

Initial call for post-quantum constructions proposals

2022

Algorithms selected

2024

First FIPS standards published

Standardized PQC Algorithms

Encryption (KEM)

- **Kyber** (ML-KEM)
- **HQC** (HQC-KEM)

Digital Signatures

- **Dilithium** (ML-DSA)
- **Falcon** (FN-DSA)
- **Sphincs+** (SLH-DSA)

Ongoing Deployment

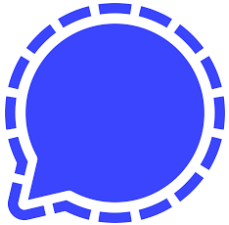
Browsers

Google Chrome

Firefox

...

Secure Messaging



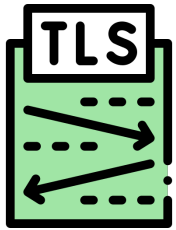
Signal

Apple iMessage

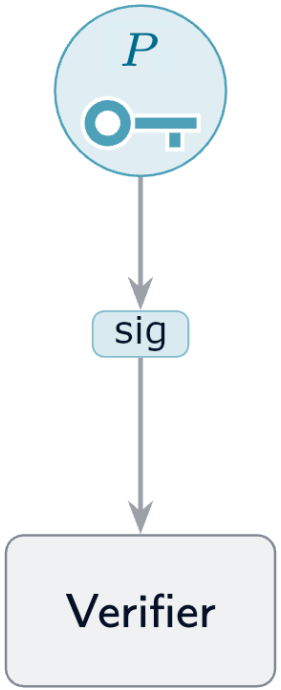
Cloud Providers

What Else Is Needed?

- **Integration Problems:** Integration in remaining existing protocols, within constraints.
  - Remaining challenges in Signal, VPNs (Wireguard), TLS authentication, etc.
- **Need For Advanced Properties:** Some applications are not satisfied with traditional signature schemes.
  - Anonymity, Group settings, Aggregation, etc ...
  - ***In this thesis:*** **High-value settings need higher key protection.**

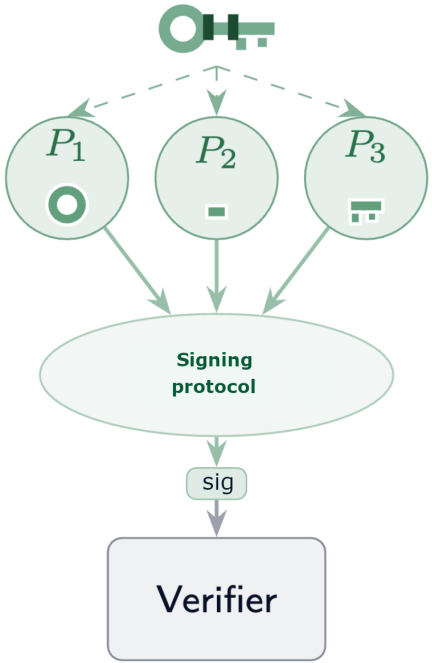


Threshold Signatures



- Generally, the signing key is stored and used from a centralized location.
  - What if *P* becomes unavailable?
  - What if *P* gets corrupted?

Threshold Signatures



- Distribute the signing key to mitigate single point of failures.

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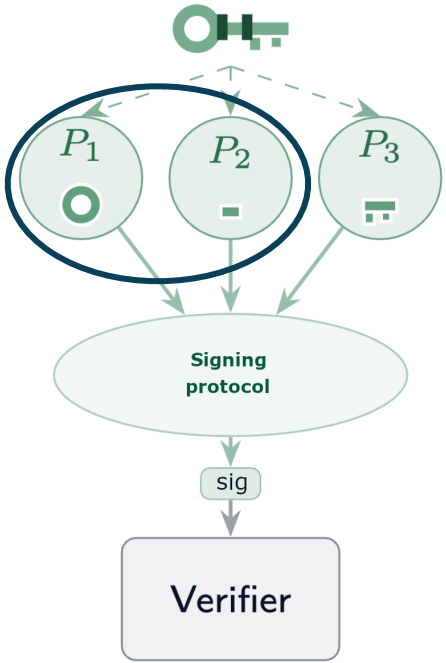

- Distribute the signing key to mitigate single point of failures.
- T-out-of-N threshold signing.
  - **Correctness:** Any set of T parties can sign.
  - **Security:** Any set of fewer than T parties cannot.

Threshold Signatures: The Security Model

- **Correctness:** Just check that the signature is valid when any T signers execute the protocol honestly!
- **Security:** *"Any set of fewer than T parties cannot sign."*

Corruption model of up to $T - 1$ parties?

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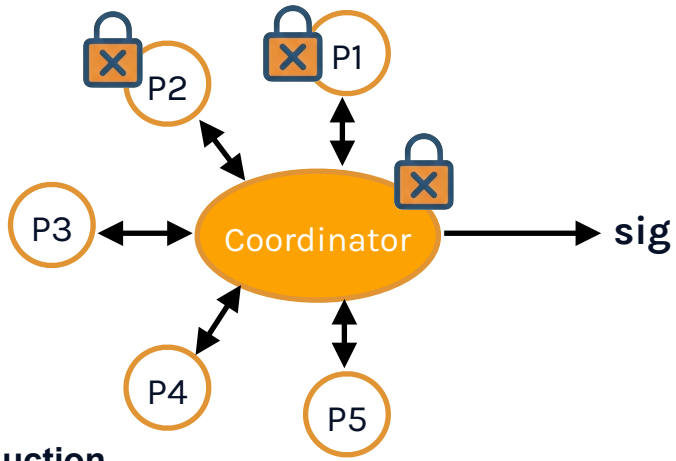
What does it mean to be unable to produce a signature?



- We consider active attackers and a malicious coordinator.
- We count a message as signed as soon as one honest signer accepts it.

Threshold Signatures: The Classical Setting

In the classical setting, we have mature threshold solutions for all main standards:

EdDSA	ECDSA	BLS	RSA
FROST [KG20]	[CGGMP21]	[Bol03]	[Sho00]

Threshold Signatures: The PQ Standards?

It is desirable to design threshold solutions for the new NIST PQ standards.

ML-DSA

(Dilithium)

FN-DSA

(Falcon)

SLH-DSA

(SPHINCS+)

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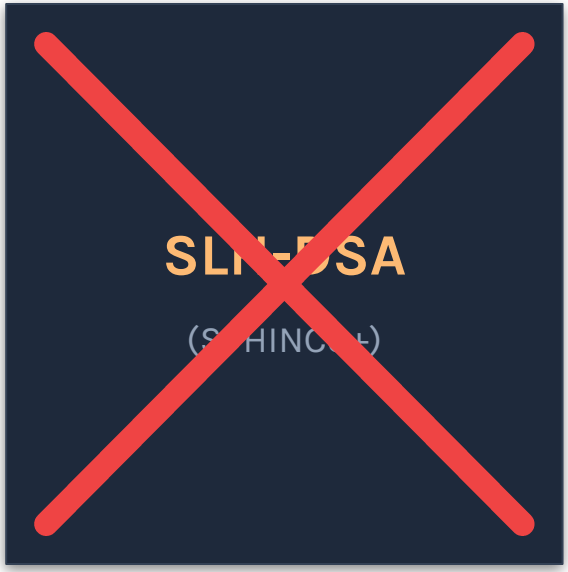
Operations too complex to distribute efficiently (e.g. floating point)



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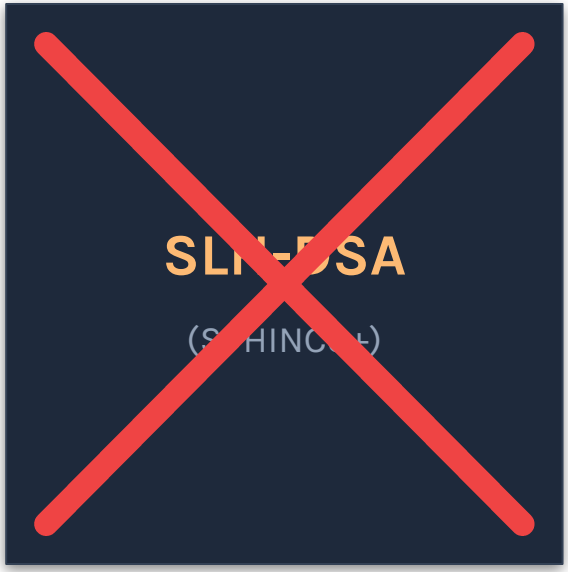

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Threshold Signatures: The PQ Standards?

ML-DSA

(Dilithium)

Possible with heavy techniques:

- From **MPC**: [BCEPT25], but high communication
- From **FHE**: [PPS26], but high complexity / computation cost

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The main contribution of this thesis:

Design of an efficient Threshold ML-DSA for a small number of parties (e.g. <10), without heavy techniques.

Approaches For Threshold ML-DSA

Threshold ML-DSA constructions still come with important trade-offs.

Paradigm	# parties supported	Comm./party	Computation	Detect Misbehavior
MPC	∞	**> 1 MB (> 20 rounds)**	Medium	✗
FHE	∞	25 kB (2 rounds)	**High**	✗
This thesis	**< 10**	20 kB - 1MB (3 rounds)	Low	✗

Threshold Signatures: Advanced properties?

Raccoon

Threshold-friendly signature scheme.

Although non standard and bigger signatures than ML-DSA (10 kB vs 2.5 kB).

Threshold Raccoon: Practical Threshold Signatures from Standard Lattice Assumptions

Rafael del Pino[1], Shuichi Katsumata[1,2], Mary Maller[1,3], Fabrice Mouhartem[4], Thomas Prest[1], Markku-Juhani Saarinen[1,5]

# parties supported	Comm./party	Computation	Detect Misbehavior
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Second part of this talk:

Threshold Raccoon with protection against misbehavior.

This Thesis & Presentation Roadmap

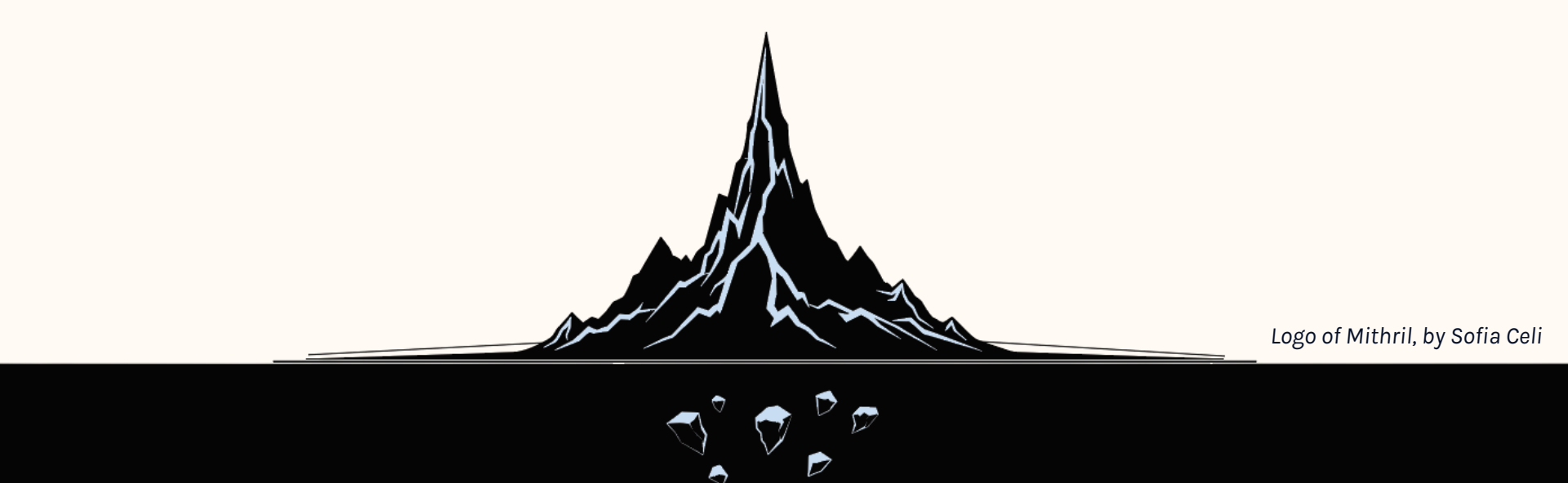
This thesis introduces three new threshold constructions answering open questions.

Scheme	Feature
Mithril	Standard compatibility
IA-Raccoon	Identifies bad behavior
RB-Raccoon	Corrects bad behavior

This talk*

* Focus on the high-level constructions, omitting proof techniques.

Part 1. Mithril: An Efficient Threshold ML-DSA



Logo of Mithril, by Sofia Celi

ML-DSA Signatures

ML-DSA . Keygen() → sk, vk

- Sample uniform $\mathbf{A}$
- $\text{vk} = (\mathbf{A}, \mathbf{A} \cdot \text{sk} + \mathbf{e})$, for sk, $\mathbf{e}$ short

MLWE **assumption:** vk appears uniformly distributed.

Signing → Fiat-Shamir transform applied to protocol proving knowledge of sk.

1. **Randomness + Commitment:** Sample short $\mathbf{r}$, and commit $\mathbf{w} = \lfloor \mathbf{A} \cdot \mathbf{r} \rceil$.
2. **Challenge:** Derive challenge $c = H(\mathbf{w}, \text{msg})$.
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Verification → Check that $\mathbf{w}$ can be recovered from $\mathbf{z}$, and $\mathbf{z}$ is short.

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2. Threshold ML-DSA

Distributing ML-DSA: Mithril At A High Level

Centralized

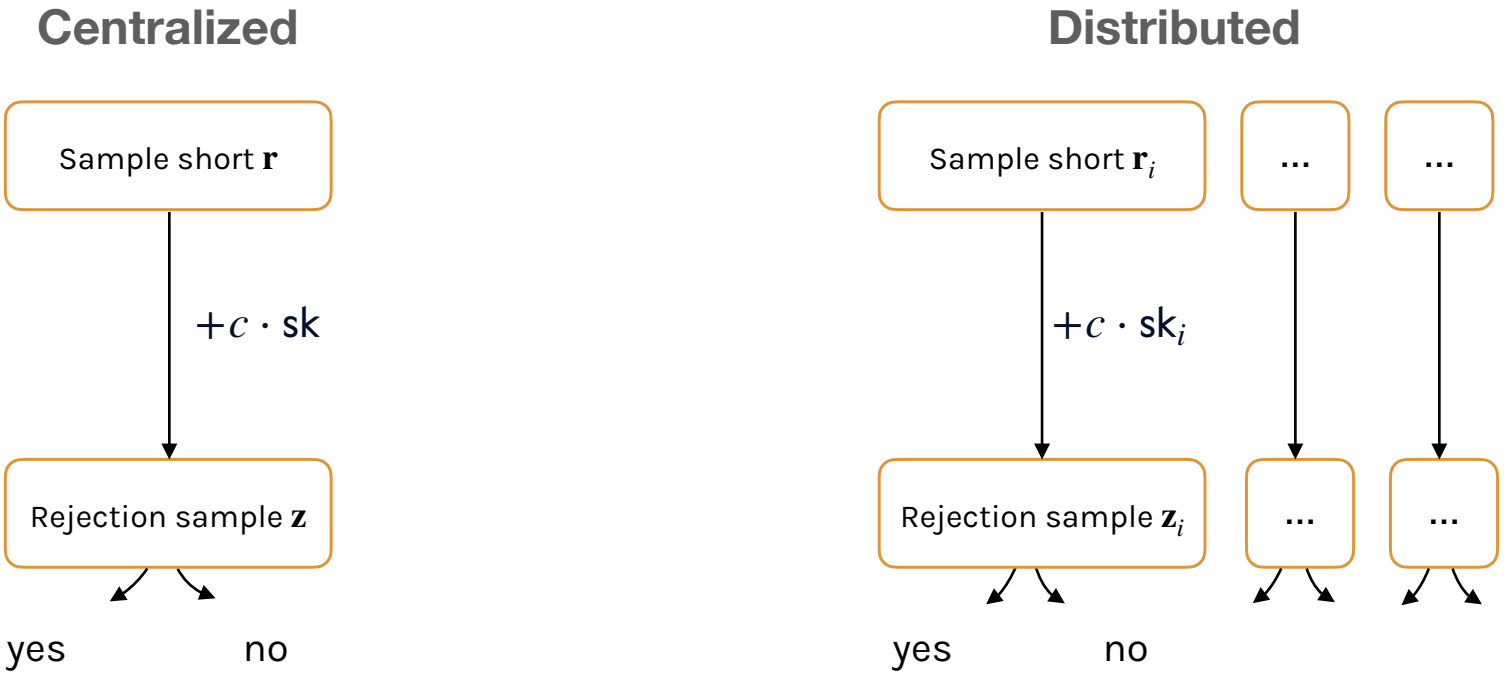
Sample short $\mathbf{r}$

Distributed

Sample short $\mathbf{r}_i$ | ... | ...

Aggregate $\mathbf{r} = \sum_i \mathbf{r}_i$

Distributing ML-DSA: Mithril At A High Level

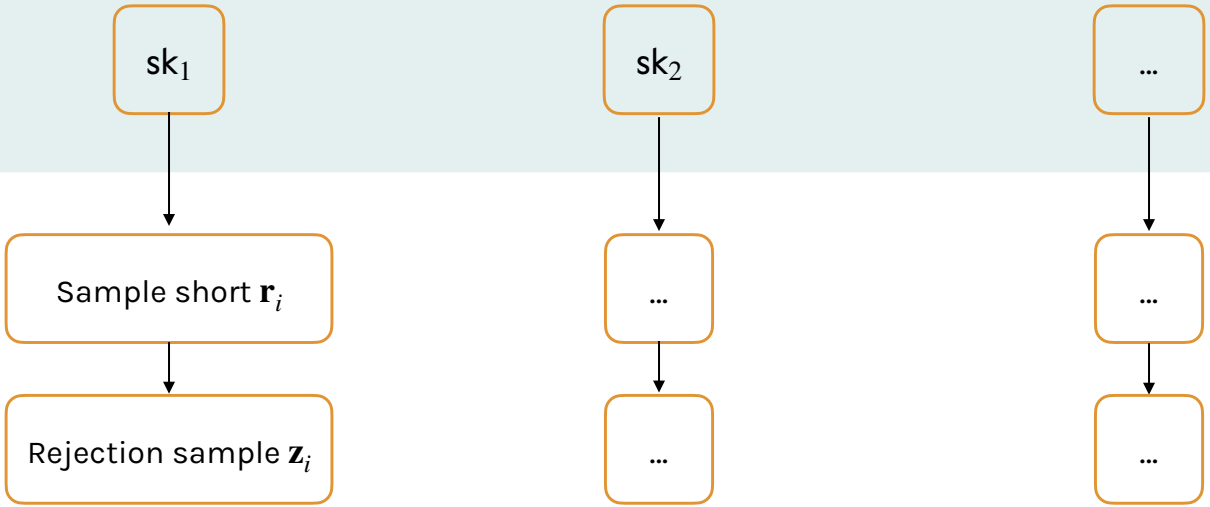


If all parties accepted, aggregate $\mathbf{z} = \sum \mathbf{z}_i$

Technique 1: (Short) Replicated Secret Sharing

For this to work, we need a **short partial secret** per party for each session.

$$\mathsf{sk} = \sum_i \mathsf{sk}_i$$



Distributing ML-DSA: Mithril At A High Level

ML-DSA signing

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2. **Challenge:** Derive challenge $c = H(\mathbf{w}, \text{msg})$.
3. **Response:** Compute $\mathbf{z} = c \cdot \text{sk} + \mathbf{r}$. Rejection sample.

Our protocol

Mithril . Sign(msg) → sig

Round 1:

1. Sample short $\mathbf{r}_i, \mathbf{e}'_i$
2. $\mathbf{w}_i = \mathbf{A} \cdot \mathbf{r}_i + \mathbf{e}'_i$
3. Broadcast $\text{commit}_i = H(\mathbf{w}_i)$

Round 2:

1. Broadcast $\mathbf{w}_i$

Round 3:

1. $\mathbf{w} = \sum_i \mathbf{w}_i$ + abort if inconsistent commit_i
2. $c = H(\lfloor \mathbf{w} \rceil, \text{msg})$
3. $\mathbf{z}_i = c \cdot \text{sk}_i + \mathbf{r}_i$
4. If $\mathbf{z}_i$ in S, **broadcast $\mathbf{z}_i$, else abort**

Combine:

1. $\text{sig} = (\sum_i \mathbf{z}_i, \lfloor \mathbf{w} \rceil)$
2. If sig not in S', **restart**
3. **return** sig

Technique 1: (Short) Replicated Secret Sharing

For this to work, we need a **short partial secret** per party for each session.

Use Replicated Secret Sharing.

ML-DSA* . Keygen() $\rightarrow$ sk, vk

1. For every possible set I of $N - T + 1$ parties
    2. $\mathsf{vk}_I = \mathbf{A} \cdot \mathsf{sk}_I + \mathbf{e}_I$, where $\mathsf{sk}_I, \mathbf{e}_I$ short
    3. Distribute $\mathsf{sk}_I, \mathbf{e}_I$ to parties in I
4. $\mathsf{vk} = \sum_i \mathsf{vk}_I$

1. When at most $T - 1$ parties are corrupted, at least one of these secrets remains hidden.
2. T parties can collaboratively reconstruct the full secret.

Partition $\sqcup_{i \in SS} m_i = \{I \text{ s.t. } |I| = N - T + 1\}$:

$$\mathsf{sk} = \sum_{i \in SS} \sum_{I \in m_i} \mathsf{sk}_I, \quad \mathbf{e} = \sum_{i \in SS} \sum_{I \in m_i} \mathbf{e}_I$$

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When T users sign → proba that all parties pass rejection sampling is p^T.

Exponential degradation over centralized setting.

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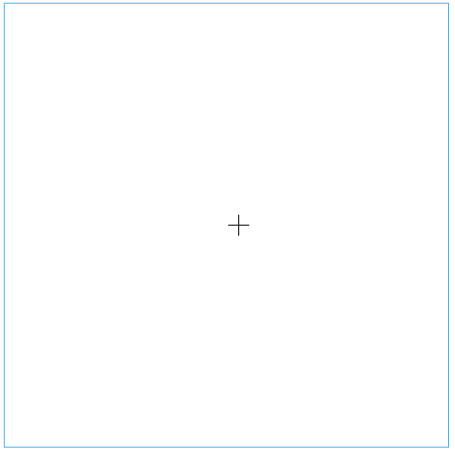

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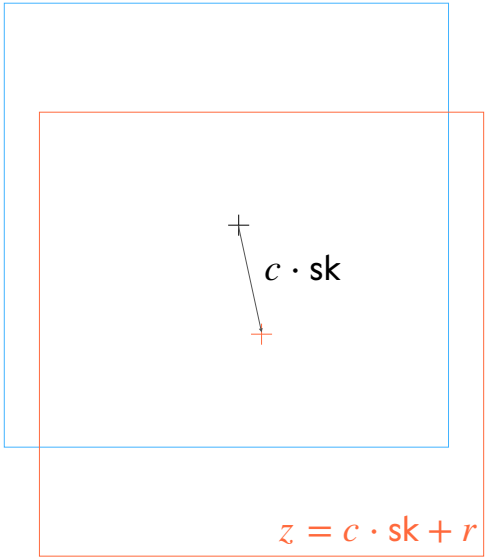

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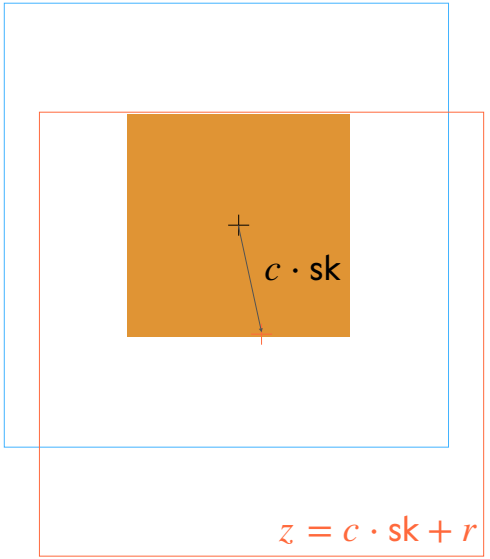
We reject any z outside of  .
The resulting distribution is independent of the secret.



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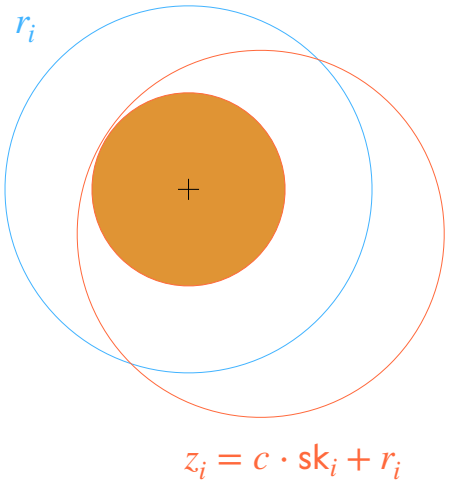

Proof Strategy

ML-DSA is proven secure in the centralized setting by replacing signatures revealed by samples independent of the secret key.

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Possible in the Random Oracle Model (ROM):

- Sample challenge c, and response $\mathbf{z}$ first (independent of secret).
- Compute consistent commitment $\mathbf{w} = \lfloor \mathbf{A} \cdot \mathbf{z} - c \cdot \mathsf{vk} \rceil$.
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This works well because rejected values (dependent on the secret) are completely discarded.

Proof Strategy

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Prior techniques:

- Left-over hash lemma / regularity lemma in [BTT22].
- Renyi divergence + search-to-decision reduction of MLWE in [DFPS23].

→ Does not apply to ML-DSA parameters.

Proof Strategy

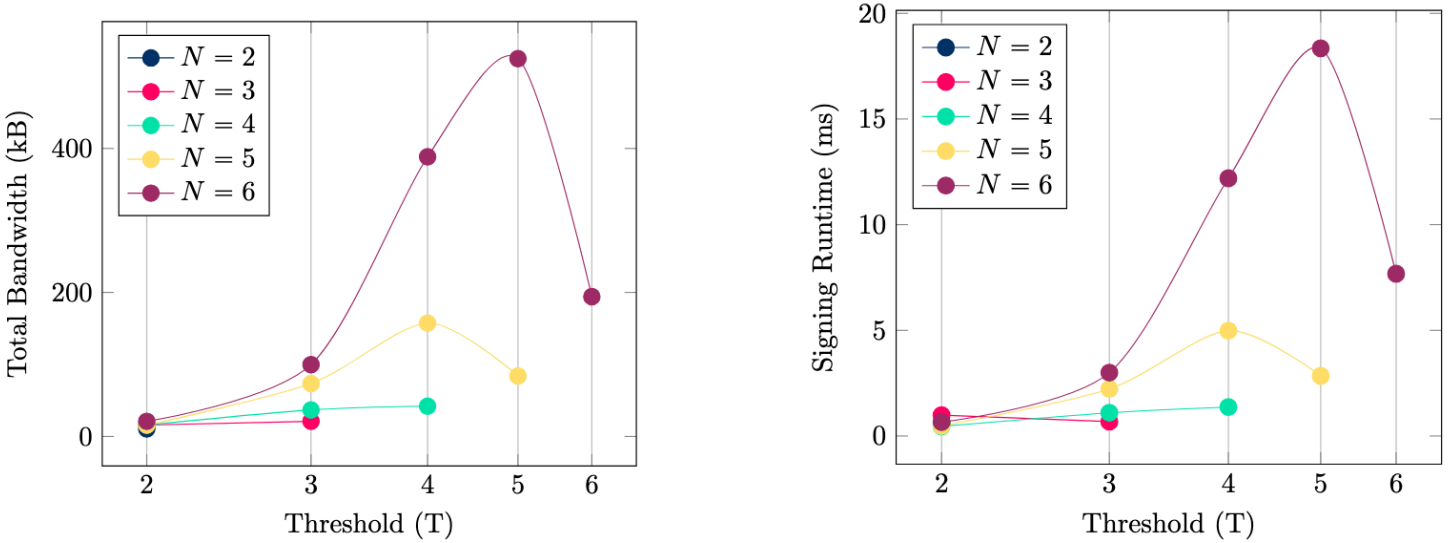
In the threshold setting, $\mathbf{w}_i$ for discarded values is still revealed.

Lemma: Rejected $\mathbf{w}_i$ is indistinguishable from uniform if:

- MLWE is hard over randomness and response distributions.

Performance: Bandwidth and latency

Parameters aim for a success probability 1/2 for each attempt (vs ~1/4 in original ML-DSA).



Bandwidth and latency of threshold signing for ML-DSA 44 (on a local network)

Parties are executed in parallel, and we average over successful attempts.

Instantiation

Scheme	Feature	Signature Size
Mithril	Standard compatibility	2.5 kB (ML-DSA)

... but no mechanism to detect misbehavior making the protocol abort.

Part 2. Advanced Properties With Threshold Raccoon

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3. Threshold Raccoon with Misbehavior Detection

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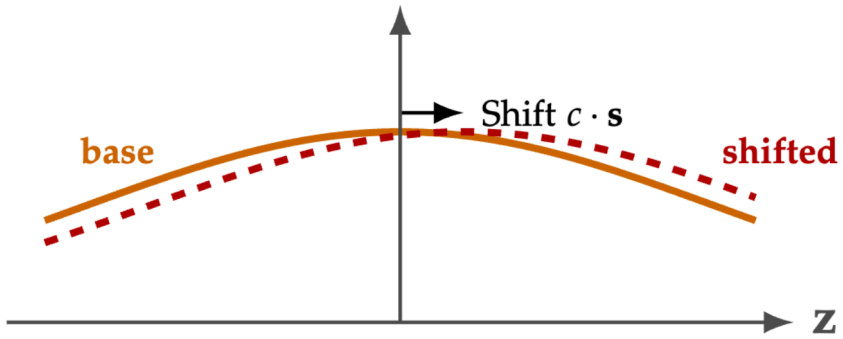
   - If $\mathbf{z} \in S$ → **output**

3. Threshold Raccoon with Misbehavior Detection

Raccoon Signatures

If $\mathbf{r}$ sufficiently large, the secret shift is hard to detect.



→ Raccoon chooses larger randomness and gets bigger signatures (10 kB vs 2.5 kB for ML-DSA).

Prior Work: Threshold Raccoon Without Misbehavior Detection

Raccoon signing

1. **Randomness + Commitment:** Sample short $\mathbf{r}, \mathbf{e}$, and commit $\mathbf{w} = \lfloor \mathbf{A} \cdot \mathbf{r} + \mathbf{e} \rceil$.
2. **Challenge:** Derive challenge $c = H(\mathbf{w}, \text{msg})$.
3. **Response:** Compute $\mathbf{z} = c \cdot \text{sk} + \mathbf{r}$. Rejection sample.

Shamir shares

Threshold Raccoon

ThRaccoon . Sign(msg) → sig

Round 1:

1. Sample short $\mathbf{r}_i, \mathbf{e}'_i$
2. $\mathbf{w}_i = \mathbf{A} \cdot \mathbf{r}_i + \mathbf{e}'_i$
3. Broadcast $\text{commit}_i = H(\mathbf{w}_i)$

Round 2:

1. Broadcast $\mathbf{w}_i$

Round 3:

1. $\mathbf{w} = \sum_i \mathbf{w}_i$ + abort if inconsistent commit_i
2. $c = H(\lfloor \mathbf{w} \rceil, \text{msg})$
3. $\mathbf{z}_i = c \cdot \lambda_i \cdot \text{sk}_i + \mathbf{r}_i + \Delta_i$, where $\Delta_i = \text{ZeroShare(transcript)}$
4. ~~If $(\mathbf{z}_i, \mathbf{y}_i)$ in S, broadcast $\mathbf{z}_i$, else abort~~

Combine:

1. $\text{sig} = (\sum_i \mathbf{z}_i, \lfloor \mathbf{w} \rceil)$
2. ~~If sig not in S', restart~~
3. return sig

3. Threshold Raccoon with Misbehavior Detection

Adding Misbehavior Detection

Raccoon signing

1. **Randomness + Commitment:** Sample short $\mathbf{r}, \mathbf{e}$, and commit $\mathbf{w} = \lfloor \mathbf{A} \cdot \mathbf{r} + \mathbf{e} \rceil$.
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Our protocol

ThRaccoon . Sign(msg) → sig

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2. ~~If sig not in S', restart~~
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3. Threshold Raccoon with Misbehavior Detection

Adding Misbehavior Detection

Can we identify malicious behavior in this protocol?

When using short shares as in Mithril, we can simply check that $\mathbf{z}_i$ is short, $\mathbf{w}_i \approx \mathbf{A} \cdot \mathbf{z}_i - c \cdot \mathsf{vk}_i$.

Our protocol

ThRaccoon . Sign(msg) → sig

Round 1:

1. Sample short $\mathbf{r}_i, \mathbf{e}'_i$
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3. Threshold Raccoon with Misbehavior Detection

Supporting More Parties?

Sadly, replicated secret sharing has $\binom{N}{T-1}$ shares. Realistically supports up to ~16 parties.

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k-out-of-N/2 (T-k)-out-of-N/2

For any set of T among N users, there will be k users on the left, $T - k$ on the right.

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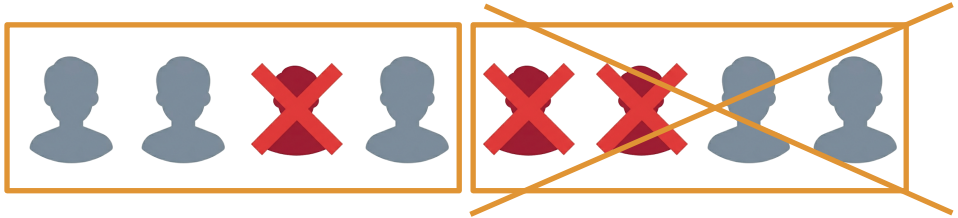

As there are at most $T-1$ corrupted parties, less than k on the left OR less than $T-k$ on the right.

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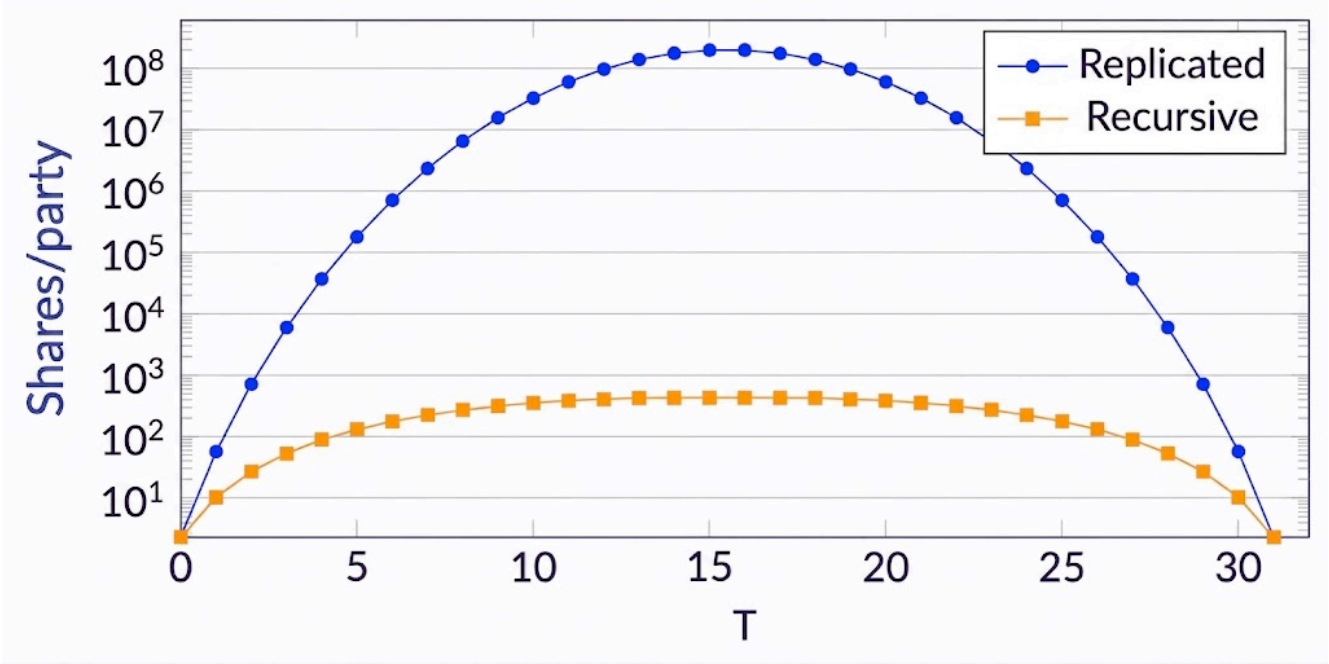
Supporting More Parties?



Number of shares for $N = 32$ parties.

3. Threshold Raccoon with Misbehavior Detection

Instantiation

Scheme	Feature	Signature Size
Mithril	Standard compatibility	2.5 kB (ML-DSA)
IA-Raccoon	Identifies bad behavior	12.3 kB

3. Threshold Raccoon with Misbehavior Detection

Threshold Raccoon: Preventing malicious behavior?

In this presentation, we developed techniques to **detect** malicious behavior.

In my thesis, also techniques to **correct** it.

- Use of Shamir Sharing + Distributed proofs of shortness for $\mathbf{r}$.
- Only one share per party → no memory blow-up.
- But,
  - This requires a gap between the security and signing threshold.
  - Larger communication, and slightly larger signature size.

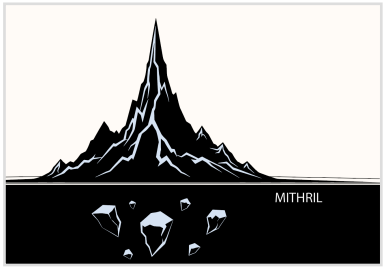
Conclusion

This thesis aimed to increase the practicality of lattice-based threshold signatures.

Scheme	Feature	Signature Size
Mithril	Standard compatibility	2.5 kB (ML-DSA)
IA-Raccoon	Identifies bad behavior	12.3 kB
RB-Raccoon	Corrects bad behavior	15.1 kB

NIST Call For Multi-Party Threshold Cryptography

Since 2019, NIST organized a few workshops to discuss threshold cryptography. In 2026, formal call for proposals: will consolidate knowledge and potentially lead to standards.



Mithril

Threshold ML-DSA

Hermine

2-round Threshold Raccoon

- *Key refresh*
- *Identifiable Aborts*

Vinaigrette

Threshold MAYO

Techniques not discussed in the manuscript.

Open Problems

- For the schemes presented here specifically:
  - Scalability to more parties of Mithril, with better secret sharing
  - Optimized implementations (memory, side-channel resistance)
  - More properties (key refresh in Mithril, a posteriori key sharing in constant time)
- Threshold schemes at large
  - DKG and stronger security models
  - Efficient solutions for blockchain consensus (requires signature aggregation)
  - Threshold variants for more NIST Additional Signature Candidates?

Questions?