The NIST MPTC Call

Post-Quantum Threshold Signatures and a Promising Approach to Threshold ML-DSA

Guilhem Niot

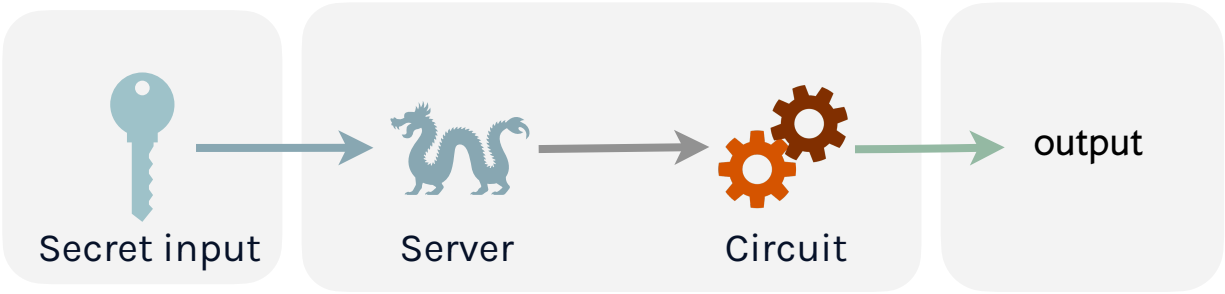
Séminaire C2 - 03/07/2026

Presentation Roadmap

I. Overview of the NIST MPTC Call, and the different dimensions considered.

II. In-depth introduction of Mithril, an efficient Threshold ML-DSA proposal.

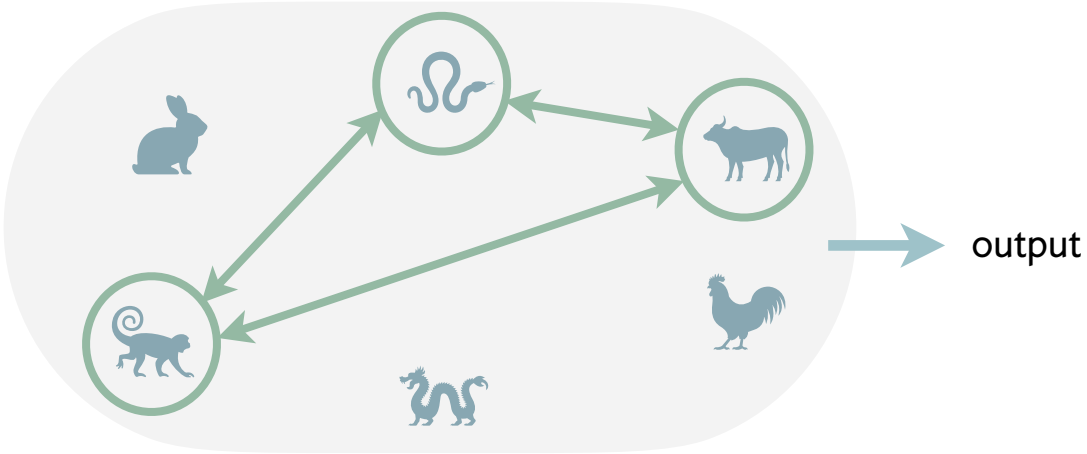
Secure Computation: The Centralized Setting



- What if the Server has limited trust? It may be malicious or hacked?
- What if the Server becomes available?

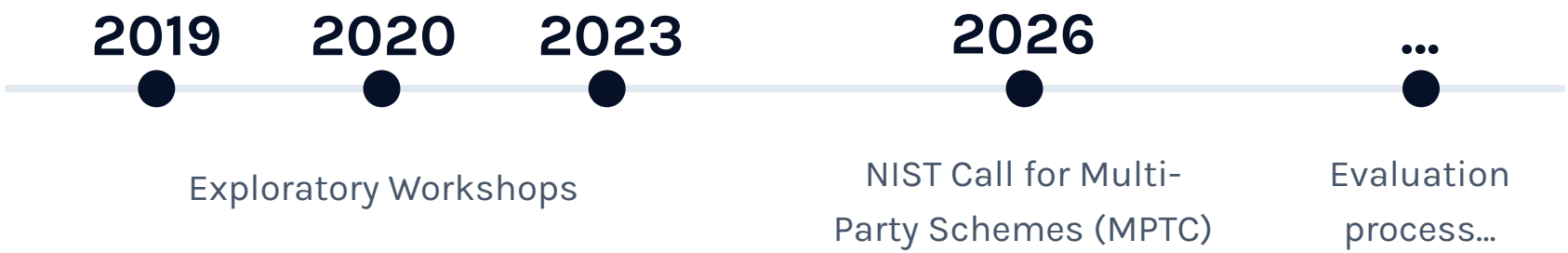
Secure Computation: The Multi-Party Setting

The secret input is shared among several parties.



They execute an interactive protocol to compute the circuit such that no single party learns the secret input.

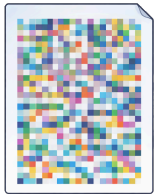
Multi-Party Cryptography and NIST



Stated goal: Gather high-quality reference material, and inform the development of future NIST recommendations and processes.

Multi-Party Cryptography and NIST: A Wide Scope

Threshold Signatures



Threshold Encryption

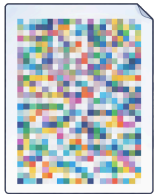
Generic techniques and Gadgets

FHE, Garbled circuits, threshold-friendly hash, etc.

Multi-Party Cryptography and NIST: A Wide Scope

Threshold Signatures

Focus of this talk



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NIST MPTC: Flexible Requirements

Interchangeability

Threshold schemes should be compatible with subsequent operations (e.g. signature verification) of a non-threshold scheme.

Primary target

Alternative base scheme

… if provides significant advantages (e.g. more efficient, can detect misbehaviour, etc.)

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Security Model

How to model user interactions? Their corruptions?

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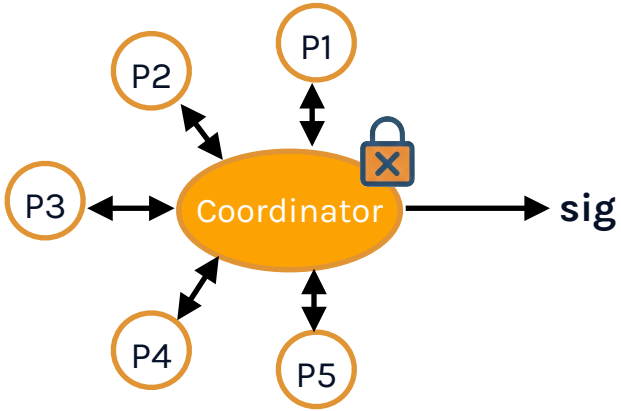
Security Model

How to model user interactions? Their corruptions?

Communication among parties is unspecified, for instance may be over (authenticated) pairwise channel, or broadcast channel.



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Corruption model: As in the literature, several corruption models exist.

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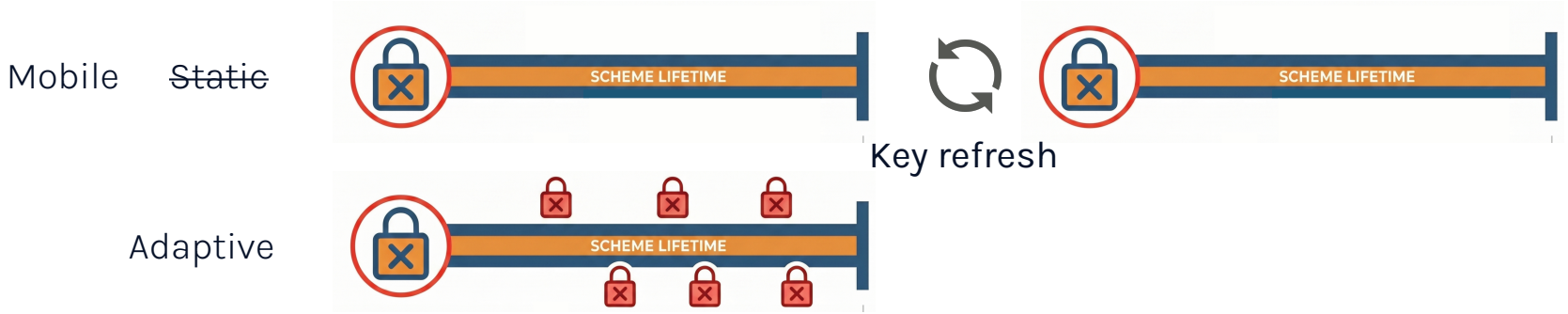

NIST MPTC: Flexible Requirements

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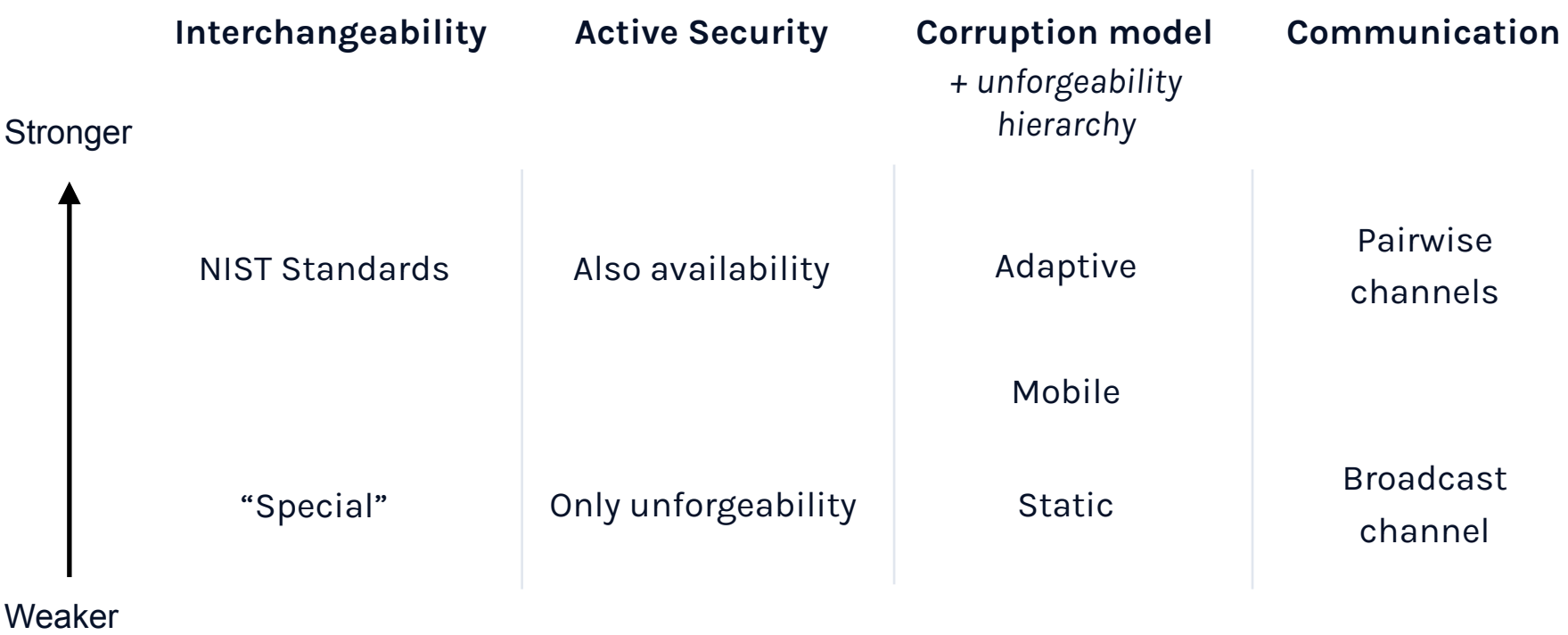
NIST requires active security (i.e. against active adversaries) for critical safety properties (i.e. unforgeability), but prevention of aborts induced by the attacker is optional.

NIST MPTC: Flexible Requirements



Threshold Signatures: The Classical Setting

In the classical setting, we have mature threshold solutions for all main standards:

EdDSA	ECDSA	BLS	RSA
FROST [KG20]	[CGGMP21]	[Bol03]	[Sho00]

Achieving adaptive security is still an active research field.
But good solutions are known for all other stronger properties mentioned.

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But good solutions are known for all other stronger properties mentioned.

Proposals submitted to NIST MPTC for EdDSA, ECDSA and BLS.

Threshold Signatures: The PQ Standards?

It is desirable to design threshold solutions for the new NIST PQ standards.

ML-DSA

(Dilithium)

FN-DSA

(Falcon)

SLH-DSA

(SPHINCS+)

Threshold Signatures: The PQ Standards?

It is desirable to design threshold solutions for the new NIST PQ standards.

PoC solution with MPC[1], but huge latency (thousands of rounds) and high communication cost.

Theoretical solution exists, but impractical due to high storage

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Threshold Signatures: The PQ Standards?

ML-DSA

(Dilithium)

There are several approaches to Threshold ML-DSA.

Paradigm	# parties supported	Comm./party	Computation	Detect Misbehavior
MPC (Quorus)	∞	**> 1 MB (> 20 rounds)**	Medium	✗
FHE[1]	∞	25 kB (2 rounds)	**High + Proprietary**	With NIZK
This talk (Mithril)	**< 10**	20 kB - 1MB (3 rounds)	Low	✗

[1]. https://eprint.iacr.org/2026/638

Threshold Signatures: The PQ Standards?

ML-DSA

(Dilithium)

- All approaches make some efficiency trade-offs.
- No satisfying way of detecting misbehaviour to prevent active attacks against availability.

Threshold Signatures: Alternative MPTC Proposals

… from other assumptions

In particular, Vinaigrette compatible with the NIST candidates MAYO/UOV.

… with state management.

Threshold variant of XMSS/LMS.

Or still from lattices, but… with high scalability, or efficient malicious behaviour detection.

Raccoon

Threshold-friendly signature scheme.
Although non standard and bigger signatures than ML-DSA (10 kB vs 2.5 kB).

Tanuki: up to 1024 parties supported.

Hermine: up to 64 parties supported, with efficient abort identification.

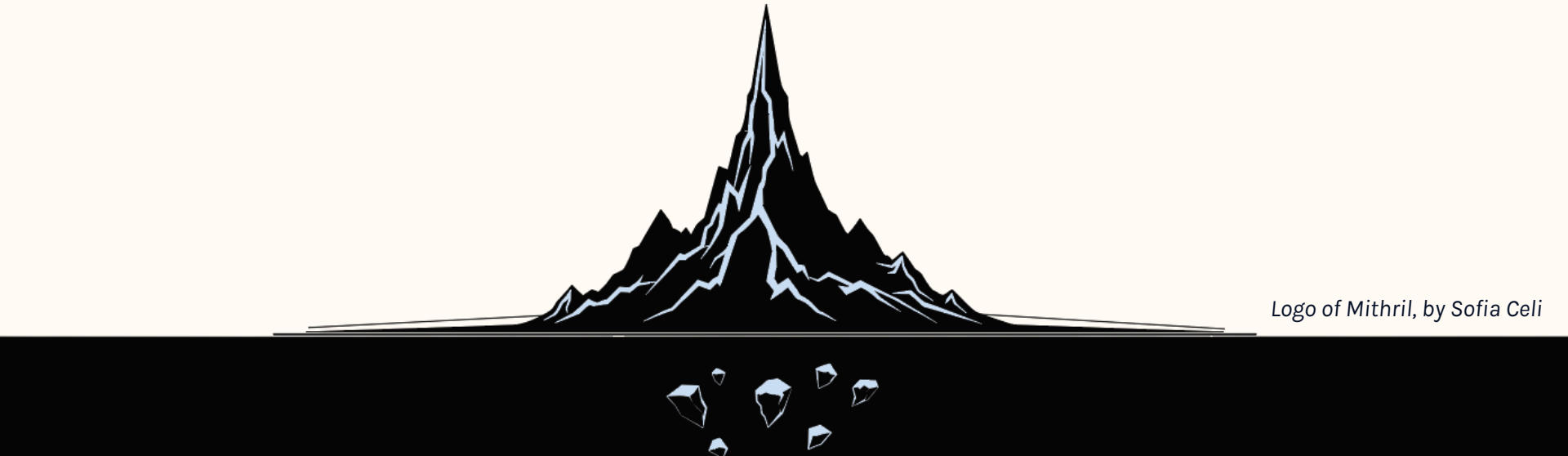
Threshold Signatures: Mithril

Mithril already attracts interest for designers who want to preserve compatibility with ML-DSA while having a favorable efficiency trade-off.

Notable use cases, with small number of parties, attacks against availability not in scope:

- Self-custody wallets
- Root of trusts

Mithril: An Efficient Threshold ML-DSA



Logo of Mithril, by Sofia Celi

ML-DSA Signatures

ML-DSA . Keygen() → sk, vk

- Sample uniform $\mathbf{A}$
- $\text{vk} = (\mathbf{A}, \mathbf{A} \cdot \text{sk} + \mathbf{e})$, for sk, $\mathbf{e}$ short

MLWE **assumption:** vk appears uniformly distributed.

Signing → Fiat-Shamir transform applied to protocol proving knowledge of sk.

1. **Randomness + Commitment:** Sample short $\mathbf{r}$, and commit $\mathbf{w} = \lfloor \mathbf{A} \cdot \mathbf{r} \rceil$.
2. **Challenge:** Derive challenge $c = H(\mathbf{w}, \text{msg})$.
3. **Response:** Compute $\mathbf{z} = c \cdot \text{sk} + \mathbf{r}$.

Verification → Check that $\mathbf{w}$ can be recovered from $\mathbf{z}$, and $\mathbf{z}$ is short.

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   - If $\mathbf{z} \notin S$ → **restart**
4. Rejection sample:
   - If $\mathbf{z} \in S$ → **output**

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2. Threshold ML-DSA

Distributing ML-DSA: Mithril At A High Level

Centralized

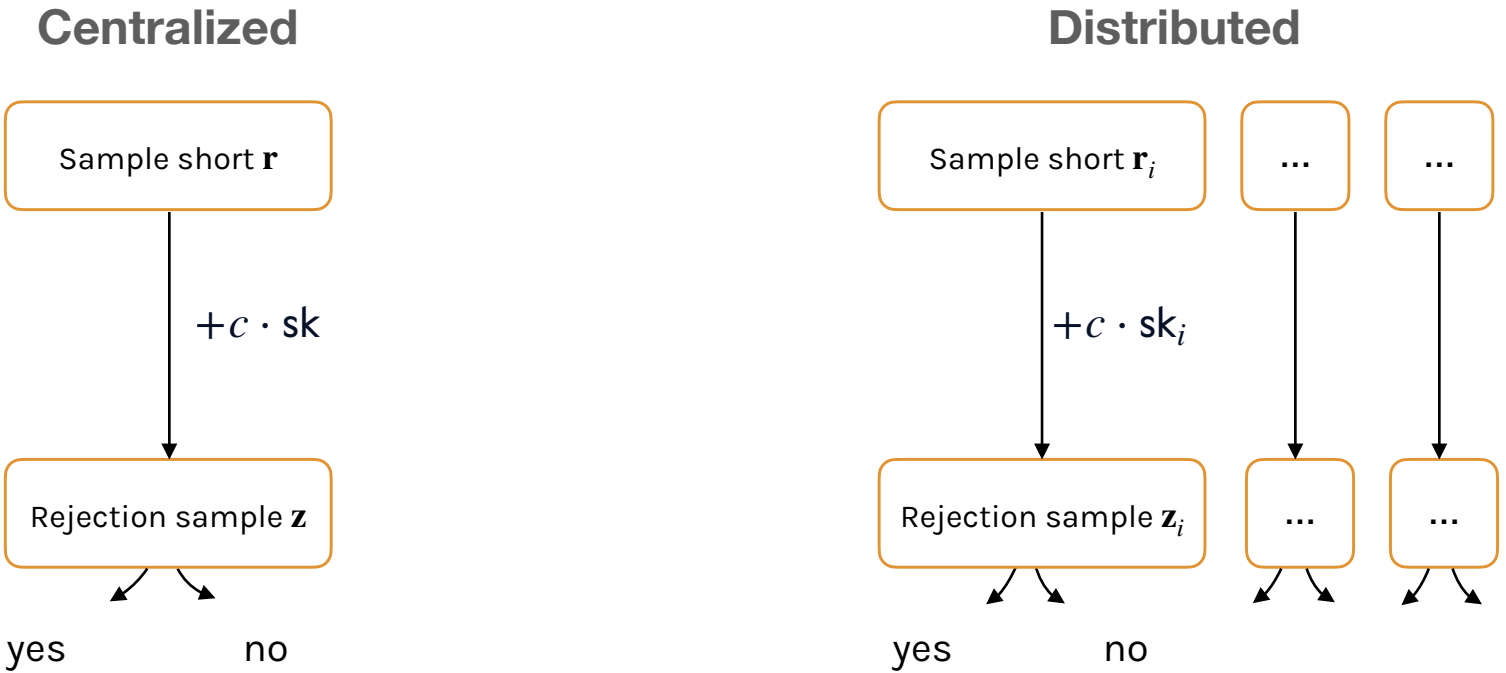
Sample short $\mathbf{r}$

Distributed

Sample short $\mathbf{r}_i$ | ... | ...

Aggregate $\mathbf{r} = \sum_i \mathbf{r}_i$

Distributing ML-DSA: Mithril At A High Level

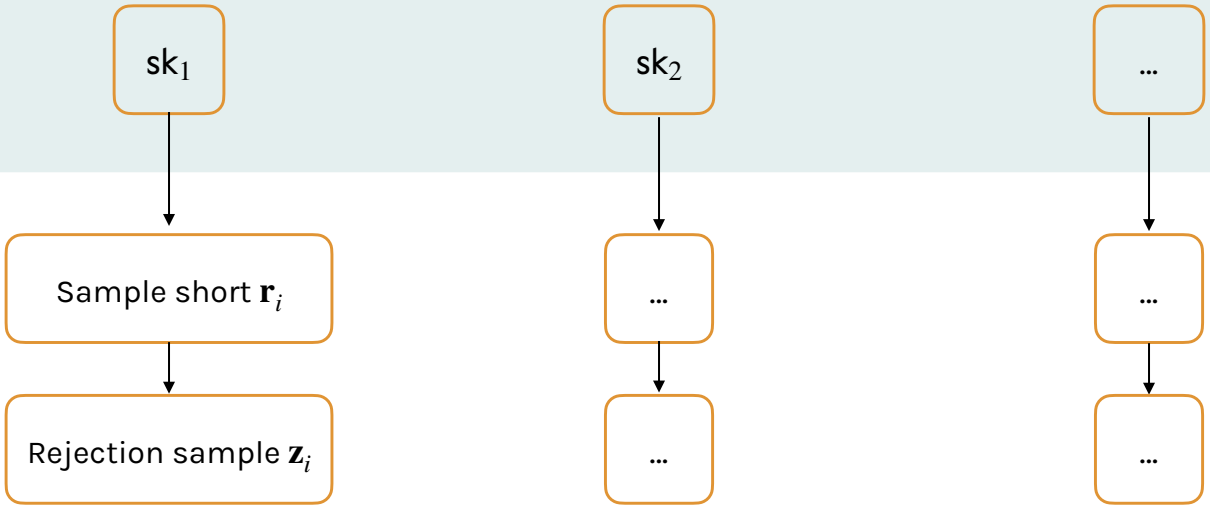


If all parties accepted, aggregate $\mathbf{z} = \sum \mathbf{z}_i$

Technique 1: (Short) Replicated Secret Sharing

For this to work, we need a **short partial secret** per party for each session.

$$\mathsf{sk} = \sum_i \mathsf{sk}_i$$



Distributing ML-DSA: Mithril At A High Level

ML-DSA signing

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2. **Challenge:** Derive challenge $c = H(\mathbf{w}, \text{msg})$.
3. **Response:** Compute $\mathbf{z} = c \cdot \text{sk} + \mathbf{r}$. Rejection sample.

Our protocol

Mithril . Sign(msg) → sig

Round 1:

1. Sample short $\mathbf{r}_i, \mathbf{e}'_i$
2. $\mathbf{w}_i = \mathbf{A} \cdot \mathbf{r}_i + \mathbf{e}'_i$
3. Broadcast $\text{commit}_i = H(\mathbf{w}_i)$

Round 2:

1. Broadcast $\mathbf{w}_i$

Round 3:

1. $\mathbf{w} = \sum_i \mathbf{w}_i$ + abort if inconsistent commit_i
2. $c = H(\lfloor \mathbf{w} \rceil, \text{msg})$
3. $\mathbf{z}_i = c \cdot \text{sk}_i + \mathbf{r}_i$
4. If $\mathbf{z}_i$ in S, **broadcast $\mathbf{z}_i$, else abort**

Combine:

1. $\text{sig} = (\sum_i \mathbf{z}_i, \lfloor \mathbf{w} \rceil)$
2. If sig not in S', **restart**
3. **return** sig

Technique 1: (Short) Replicated Secret Sharing

For this to work, we need a **short partial secret** per party for each session.

Use Replicated Secret Sharing.

ML-DSA* . Keygen() $\rightarrow$ sk, vk

1. For every possible set I of $N - T + 1$ parties
   2. $\mathsf{vk}_I = \mathbf{A} \cdot \mathsf{sk}_I + \mathbf{e}_I$, where $\mathsf{sk}_I, \mathbf{e}_I$ short
   3. Distribute $\mathsf{sk}_I, \mathbf{e}_I$ to parties in I
4. $\mathsf{vk} = \sum_i \mathsf{vk}_I$

1. When at most $T - 1$ parties are corrupted, at least one of these secrets remains hidden.
2. T parties can collaboratively reconstruct the full secret.

Partition $\sqcup_{i \in SS} m_i = \{I \text{ s.t. } |I| = N - T + 1\}$:

$$\mathsf{sk} = \sum_{i \in SS} \sum_{I \in m_i} \mathsf{sk}_I, \quad \mathbf{e} = \sum_{i \in SS} \sum_{I \in m_i} \mathbf{e}_I$$

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When T users sign → proba that all parties pass rejection sampling is p^T.

Exponential degradation over centralized setting.

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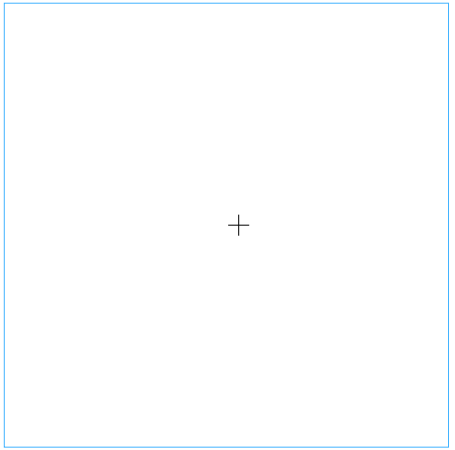

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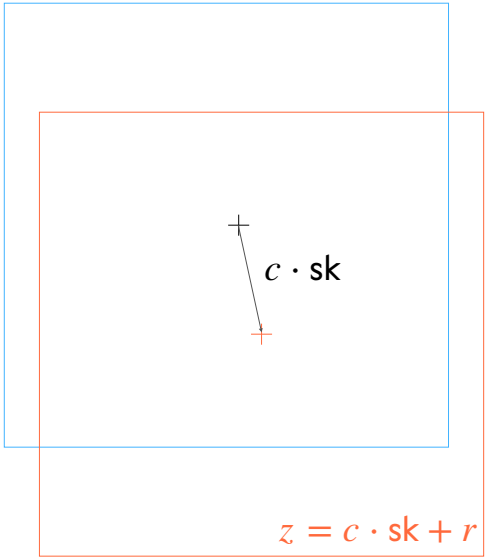

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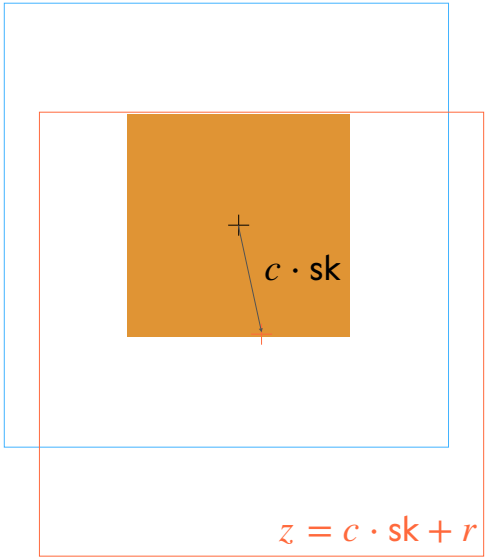
We reject any z outside of  .
The resulting distribution is independent of the secret.



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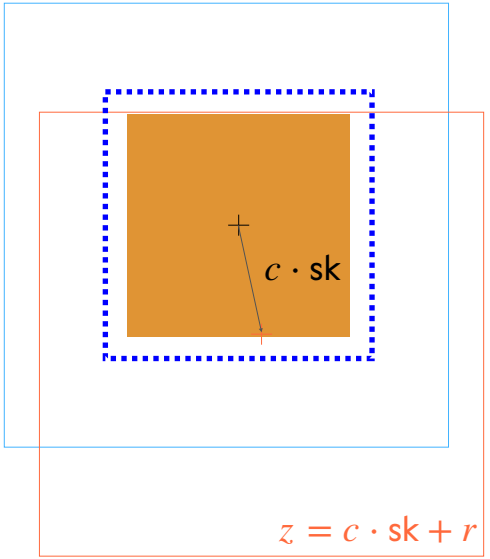

Technique 2: Optimized Rejection Sampling

When T users sign → proba that all parties pass rejection sampling is p^T.

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Sample r from a **Gaussian** distribution.

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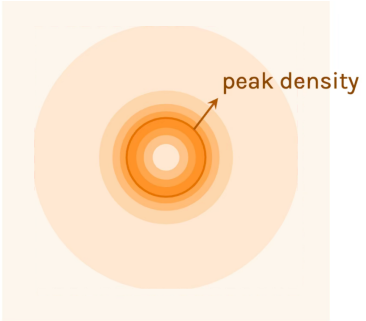
We rejection sampling $\mathbf{z}$ so that it is distributed according to a centered Gaussian independent of the secret.



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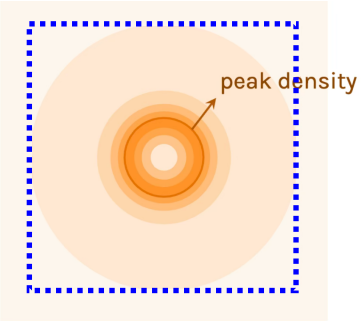

Technique 2: Optimized Rejection Sampling

1. We can accept a somewhat low success probability by performing K attempts in parallel.

2. **Unbalanced constraints:** The aggregated signature must be small enough for ML-DSA verification.
   - For the first half $\mathbf{z}$: infinite norm constraint
   - For the second half $\mathbf{y}$ + rounding: (smaller) infinite norm constraint + deserialization constraint for the recovery of $\lfloor \mathbf{w} \rceil$

   → stronger constraint on second half: we want to use smaller $\mathbf{y}$ than $\mathbf{z}$

3. The size of the Gaussian used is **proportional to the norm of the partial secret** to hide: we minimize the number of secrets used by each party in a session.

Technique 2: Optimized Rejection Sampling

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The size of the Gaussian used is **proportional to the norm of the partial secret** to hide: we minimize the number of secrets used by each party in a session.

$\binom{N}{N-T+1}$ secrets to partition among T parties.

Ideally, at most $\left\lceil \binom{N}{N-T+1} \Big/ T \right\rceil$ secrets each.

Technique 2: Optimized Rejection Sampling

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The size of the Gaussian used is **proportional to the norm of the partial secret** to hide: we minimize the number of secrets used by each party in a session.

$\binom{N}{N-T+1}$ secrets to partition among T parties.

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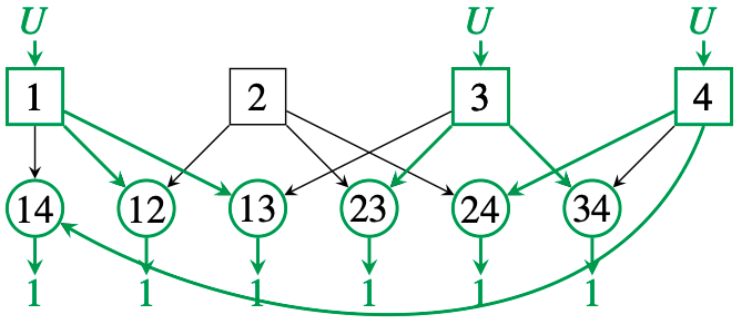
We find an optimal partition with a max-flow algorithm.



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ML-DSA is proven secure in the centralized setting by replacing signatures revealed by samples independent of the secret key.

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Possible in the Random Oracle Model (ROM):

- Sample challenge c, and response $\mathbf{z}$ first (independent of secret).
- Compute consistent commitment $\mathbf{w} = \lfloor \mathbf{A} \cdot \mathbf{z} - c \cdot \mathsf{vk} \rceil$.
- Program random oracle $H(\mathbf{w}, \mathsf{msg}) := c$.

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This works well because rejected values (dependent on the secret) are completely discarded.

Proof Strategy

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Prior techniques:

- Left-over hash lemma / regularity lemma in [BTT22].
- Renyi divergence + search-to-decision reduction of MLWE in [DFPS23].

→ Does not apply to ML-DSA parameters.

Proof Strategy

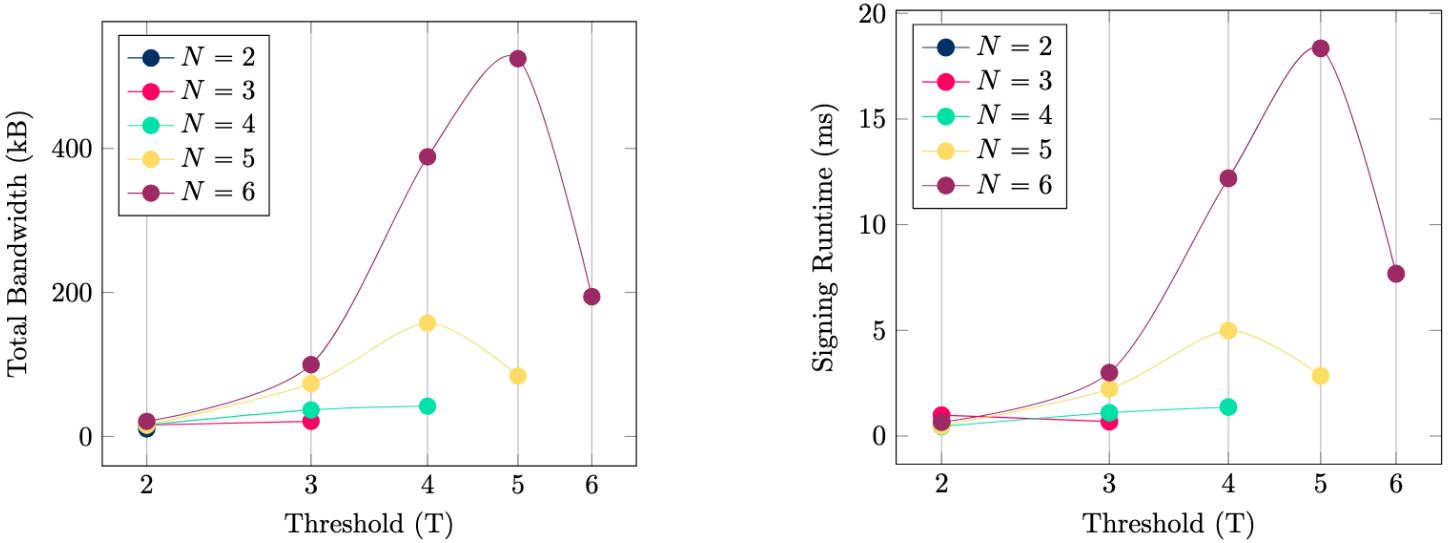
In the threshold setting, $\mathbf{w}_i$ for discarded values is still revealed.

Lemma: Rejected $\mathbf{w}_i$ is indistinguishable from uniform if:

- MLWE is hard over randomness and response distributions.

Performance: Bandwidth and latency

Parameters aim for a success probability 1/2 for each attempt (vs ~1/4 in original ML-DSA).



Bandwidth and latency of threshold signing for ML-DSA 44 (on a local network)

Parties are executed in parallel, and we average over successful attempts.

Performance: WAN latency

T=2, N=6

Latency of threshold signing for ML-DSA 44.
Parties are executed in parallel, and we average over successful attempts.

Performance: WAN latency

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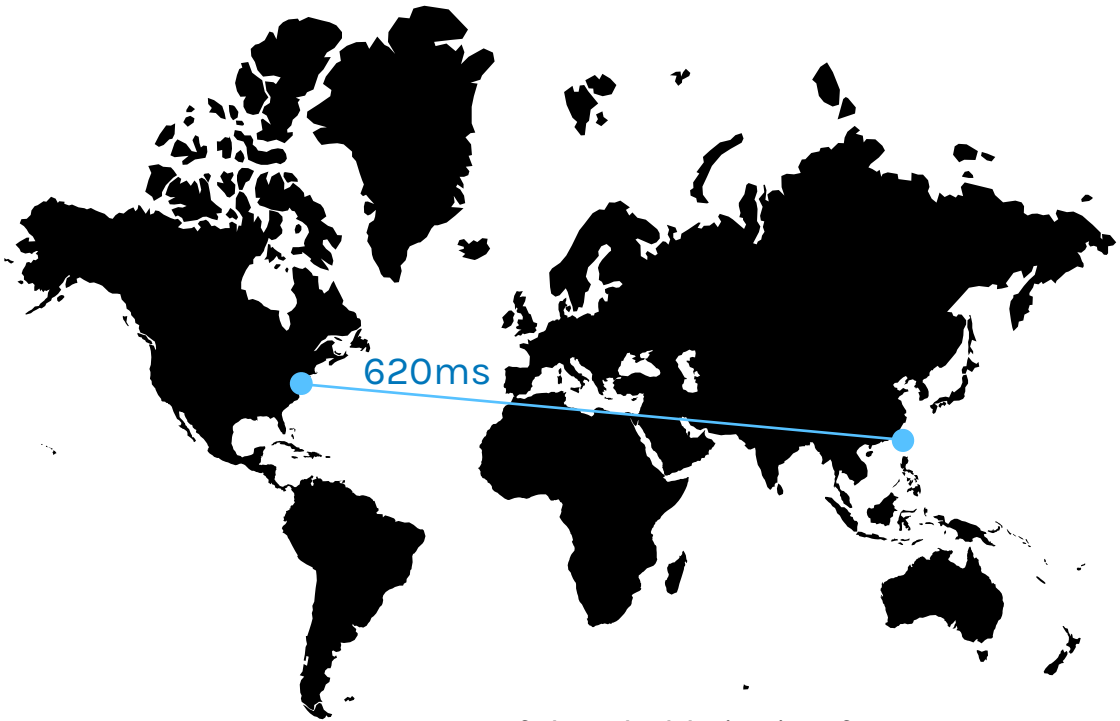

Latency of threshold signing for ML-DSA 44.
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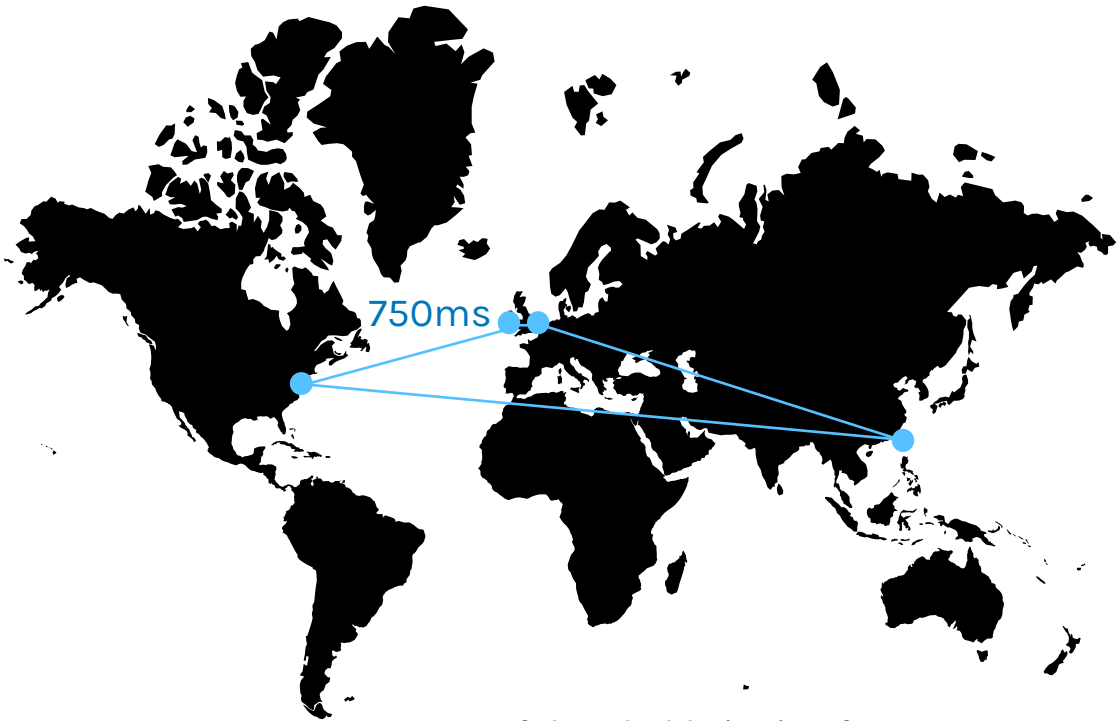
Performance: WAN latency

T=4, N=6



Latency of threshold signing for ML-DSA 44.
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Threshold ML-DSA

Scheme	Feature	Signature Size
Mithril	Standard compatibility	2.5 kB (ML-DSA)

- Signing protocol in 3 rounds.
- Support Distributed Key Generation.
- Communication between 20kB and 1MB for *a few parties*.

Open Problems

- For Mithril:
  - Formal verification
  - Optimized implementations (memory, side-channel resistance)
  - More properties (key refresh in Mithril, a posteriori key sharing in constant time)
- Threshold schemes at large
  - DKG and stronger security models
  - Efficient solutions for blockchain consensus (requires signature aggregation)
  - Threshold variants for more NIST Additional Signature Candidates?

Questions?

Bonus: Distributed Key Generation

ML-DSA* . Keygen() → sk, vk

- For every possible set I of $N - T + 1$ parties
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  - Distribute $\mathsf{sk}_I, \mathbf{e}_I$ to parties in I
- $\mathsf{vk} = \sum_i \mathsf{vk}_I$

Core idea: parties in each group I will agree on a shared secret $\mathsf{sk}_I, \mathbf{e}_I$.

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- $\mathsf{vk} = \sum_i \mathsf{vk}_I$

Core idea: parties in each group I will agree on a shared secret $\mathsf{sk}_I, \mathbf{e}_I$.

Challenge: we rely on the honest sampling of secrets to bound the norm of several secrets.

→ We can rely on the ROM to enforce honest sampling.

Bonus: Distributed Key Generation

ML-DSA*.Keygen() → sk, vk

- For every possible set I of $N - T + 1$ parties
  - $\mathsf{vk}_I = \mathbf{A} \cdot \mathsf{sk}_I + \mathbf{e}_I$, where $\mathsf{sk}_I, \mathbf{e}_I$ short
  - Distribute $\mathsf{sk}_I, \mathbf{e}_I$ to parties in I
- $\mathsf{vk} = \sum_i \mathsf{vk}_I$

Core idea: parties in each group I will agree on a shared secret $\mathsf{sk}_I, \mathbf{e}_I$.

Challenge: we rely on the honest sampling of secrets to bound the norm of several secrets.

→ We can rely on the ROM to enforce honest sampling.

Round 1: parties agree on a shared secret for each group I (ensure secrecy).

Round 1-2: Parties agree on a global **seed** (ensure honest sampling)

At the end of round 2, parties derive secrets as $\mathsf{sk}_I, \mathbf{e}_I = H(\mathsf{ss}_I, \mathsf{seed}, I)$

Round 3-4: Parties reveal partial public keys $\mathsf{vk}_I = \mathbf{A} \cdot \mathsf{sk}_I + \mathbf{e}_I$, and aggregate them in the final key.