Finally!

A Compact Lattice-Based Threshold Signature

Guilhem Niot, joint work with *Rafael del Pino*

Journées C2 2025 - 03/04/2025

1. Background

Threshold cryptography

Start with two observations…

Devices can be **compromised** or **made out of order.**

Security issue

Availability issue

Solution: share secret

Solution: replicate secret

Threshold Cryptography: T-out-of-N scheme

- T out of N parties can perform an operation
- Less than T cannot

NIST Call for Threshold Schemes

PUBLICATIONS

NIST IR 8214C (2nd Public Draft)

NIST First Call for Multi-Party Threshold Schemes

Date Published: March 27, 2025
Comments Due: April 30, 2025
Email Comments to: nistir-8214C-comments@nist.gov

Author(s)

Luís T. A. N. Brandão (NIST, Strativia), Rene Peralta (NIST)

Announcement

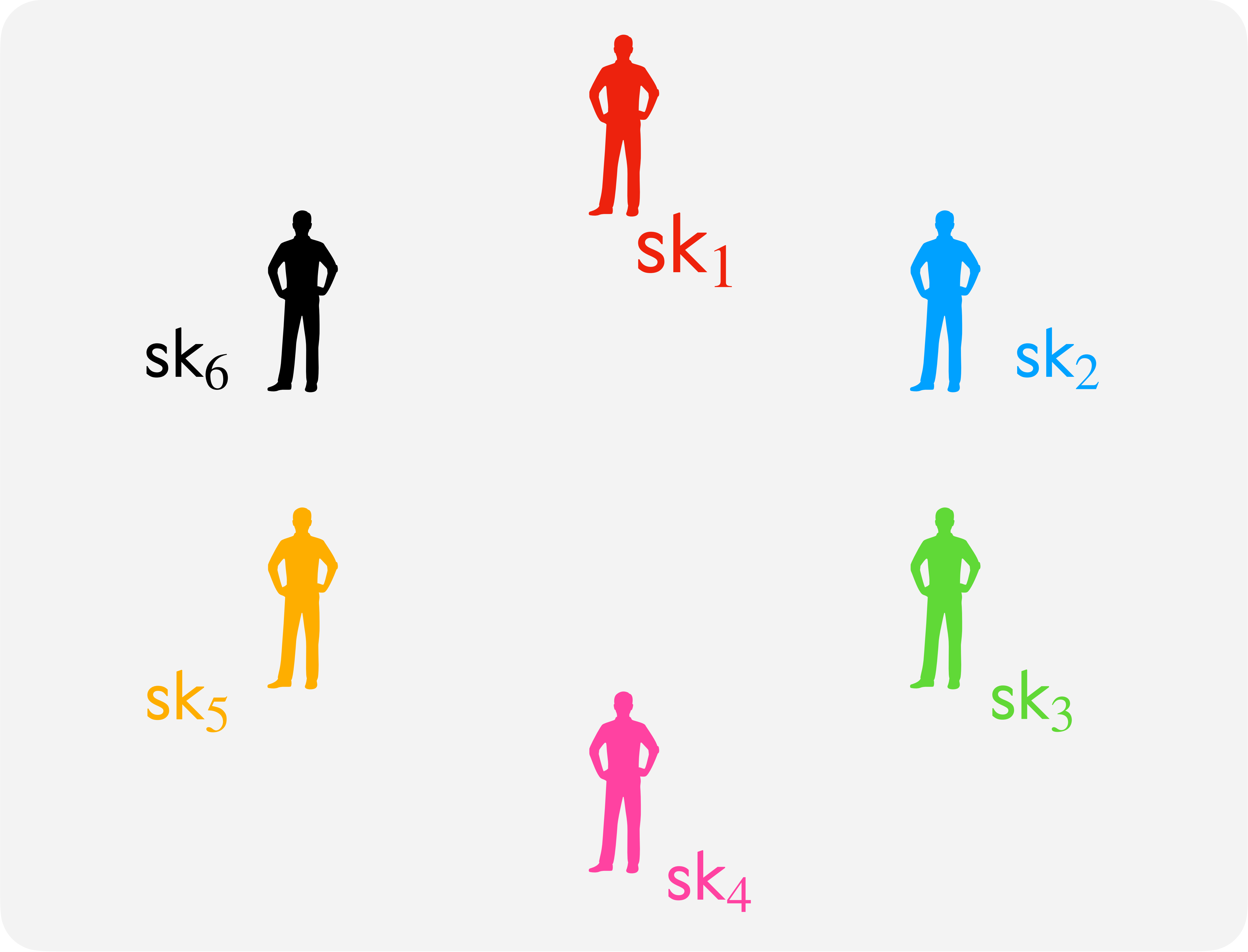
This is a second public draft. Threshold schemes should NOT be submitted until the final version of this report is published. However, the present draft can be used as a baseline to prepare for future submissions.

The scope of the call is organized into categories related to signing (Sign), public-key encryption (PKE), symmetric-key cryptography and hashing (Symm), key generation (KeyGen), fully homomorphic encryption

(T-out-of-N) threshold signatures

What are they?

An interactive protocol to distribute signature generation.

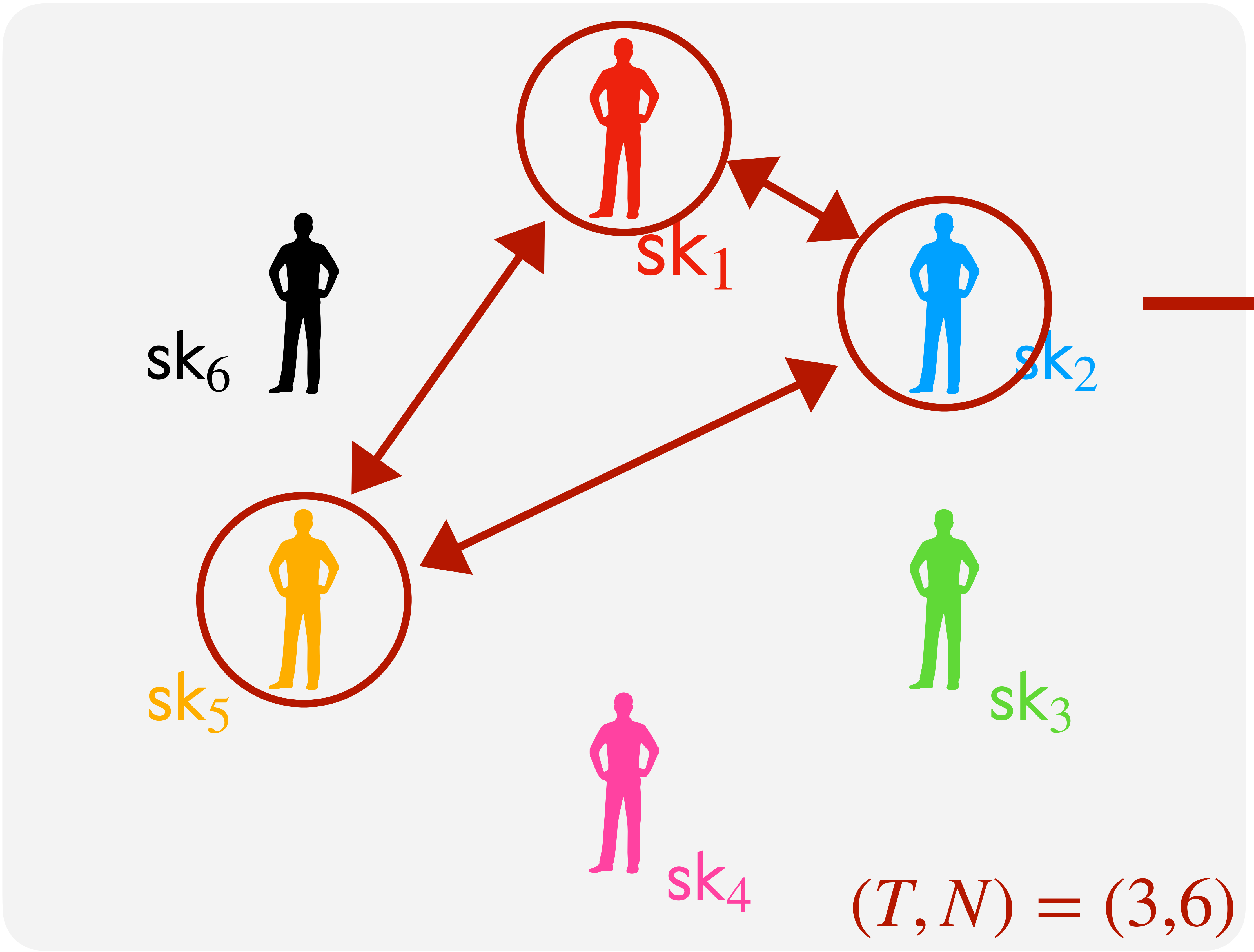


- Global verification key vk
- 1 partial signing key sk_i per party
- T-out-of-N:
  - **Correctness:** Any T out of N parties can collaborate to sign a message under vk.
  - **Unforgeability:** $T-1$ corrupted parties cannot sign.

(T-out-of-N) threshold signatures

What are they?

An interactive protocol to distribute signature generation.



Signature σ on msg

Pre-quantum solutions

- Mature solutions:
  - EdDSA: FROST [KG20]
  - ECDSA: [ANOS+21]
  - BLS: [Bol03]
  - RSA: [Sho00]

- Provide all desirable properties.

Lattice-based Threshold Signatures

An active field of research.

Threshold Raccoon: Practical Threshold Signatures from Standard Lattice Assumptions

Rafael del Pino[1], Shuichi Katsumata[1,2], Mary Maller[1,3], Fabrice Mouhartem[4], Thomas Prest[1], Markku-Juhani Saarinen[1,5]

Two-Round Threshold Signature from Algebraic One-More Learning with Errors

Thomas Espitau[1], Shuichi Katsumata[1,2], Kaoru Takemure[*] [1,2]

Ringtail: **Practical Two-Round Threshold Signatures from Learning with Errors**

Cecilia Boschini, *ETH Zürich, Switzerland*
Darya Kaviani, *UC Berkeley, USA*
Russell W. F. Lai, *Aalto University, Finland*
Giulio Malavolta, *Bocconi University, Italy*
Akira Takahashi, *JPMorgan AI Research & AlgoCRYPT CoE, USA*
Mehdi Tibouchi, *NTT Social Informatics Laboratories, Japan*

***Flood and Submerse*: Distributed Key Generation and Robust Threshold Signature from Lattices**

Thomas Espitau[1], Guilhem Niot[1,2], and Thomas Prest[1]

Two-round n-out-of-n and Multi-Signatures and Trapdoor Commitment from Lattices[⋆]

Ivan Damgård[1], Claudio Orlandi[1], Akira Takahashi[1], and Mehdi Tibouchi[2]

MuSig-L: Lattice-Based Multi-Signature With Single-Round Online Phase[⋆]

Cecilia Boschini[1], Akira Takahashi[2], and Mehdi Tibouchi[3]

Two-Round Threshold Lattice-Based Signatures from Threshold Homomorphic Encryption[⋆]

Kamil Doruk Gur[1], Jonathan Katz[2⋆⋆], and Tjerand Silde[3⋆⋆⋆]

Threshold Raccoon, a practical threshold signature

Threshold Raccoon: Practical Threshold Signatures from Standard Lattice Assumptions

Rafael del Pino[1], Shuichi Katsumata[1,2], Mary Maller[1,3], Fabrice Mouhartem[4], Thomas Prest[1], Markku-Juhani Saarinen[1,5]

Speed	Rounds	max N	| vk |	| sig |	Total communication
Fast	3	1024	4 kB	13 kB	40 kB

Designing a threshold scheme

Lattice-based Threshold Signatures

Candidate schemes

	Hash & Sign	Fiat-Shamir
Gaussian Sampling	Eagle [YJW23]	G+G [DPS23]
Rejection Sampling	Phoenix [JRS24]	Dilithium [LDK+22]
Noise Flooding	Plover [EEN+24]	Raccoon [dEK+24]

Easier to thresholdize ↓

More compact ↑

Lattice-based Threshold Signatures

Candidate schemes

Easier to thresholdize

	Hash & Sign	Fiat-Shamir
Gaussian Sampling	Eagle [YJW23]	G+G [DPS23]
Rejection Sampling	Phoenix [JRS24]	Dilithium [LDK+22]
Noise Flooding	Plover [EEN+24]	Raccoon [dEK+24]

More compact

This talk: Dilithium threshold variant.

Lattice-based Threshold Signatures

An active field of research, with different designs.

Thresholdization technique	Size	Speed	Rounds	Comm/party
MPC	S	Slow	15	≥ 1MB
FHE	M	As fast as FHE	2	≥ 1MB
Tailored	S-M	Fast	2-4	20 kB $\rightarrow 56T$ kB

Lattice-based Threshold Signatures

An active field of research, with different designs.

Thresholdization technique	Size	Speed	Rounds	Comm/party
MPC	S	Slow	15	≥ 1MB
FHE	M	As fast as FHE	2	≥ 1MB
Tailored	S-M	Fast	2-4	20 kB $\rightarrow 56T$ kB

This talk: Tailored

Two-round n-out-of-n and Multi-Si Dilithium-like
Trapdoor Commitment from Lattices*

Ivan Damgård[1], Claudio Orlandi[1], Akira Takahashi[1], and Mehdi Tibouchi[2]

$\rightarrow$ more compact and T-out-of-N?

2. Compact Dilithium-like Threshold Signatures

Finally! A Compact Lattice-Based Threshold Signature

Rafael del Pino[1] and Guilhem Niot[1,2]

Fiat-Shamir with Aborts signature

Rej$(\mathbf{v}, \chi_r, \chi_z, M) \to \mathbf{z} \mid \perp$

- $\mathbf{r} \leftarrow \chi_{\mathbf{r}}$
- $\mathbf{z} = \mathbf{v} + \mathbf{r}$
- $b \leftarrow \mathscr{B}\left(\max\left(\dfrac{\chi_{\mathbf{z}}(\mathbf{z})}{M\chi_{\mathbf{r}}(\mathbf{r})}, 1\right)\right)$
- If $b = 0$ then $\mathbf{z} = \perp$
- Return $\mathbf{z}$

Ideal$(\chi_z, M) \to \mathbf{z} \mid \perp$

- $\mathbf{z} \leftarrow \chi_{\mathbf{z}}$
- $b \leftarrow \mathscr{B}\left(\dfrac{1}{M}\right)$
- If $b = 0$ then $\mathbf{z} = \perp$
- Return $\mathbf{z}$

For proper parameters, Rej$(\mathbf{v}, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M) \sim$ Ideal$(\chi_{\mathbf{z}}, M)$.

$\to$ distribution of $\mathbf{z}$ is independent of the secret value $\mathbf{v}$

Fiat-Shamir with Aborts signature

$\mathsf{Rej}(\mathbf{v}, \chi_r, \chi_z, M; \mathbf{r}) \to \mathbf{z} \mid \perp$

- $\mathbf{z} = \mathbf{v} + \mathbf{r}$
- $b \leftarrow \mathscr{B}\left(\max\left(\frac{\chi_{\mathbf{z}}(\mathbf{z})}{M\chi_{\mathbf{r}}(\mathbf{r})}, 1\right)\right)$
- If $b = 0$ then $\mathbf{z} = \perp$
- Return $\mathbf{z}$

$\mathsf{FSwA}.\mathsf{Sign}(\mathsf{sk}, \mathsf{msg}) \to \mathsf{sig}$

- $\mathbf{r} \leftarrow \chi_{\mathbf{r}}$
- $\mathbf{w} = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}$
- $c = H(\mathbf{w}, \mathsf{msg})$
- $\mathbf{z} = \mathsf{Rej}(c \cdot \mathsf{sk}, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M; \mathbf{r})$
- If $\mathbf{z} = \perp$ then **restart**
- Return $(c, \mathbf{z})$

$\mathsf{FSwA}.\mathsf{Verify}(\mathsf{vk}, \mathsf{msg}, \mathsf{sig} = (c, \mathbf{z}))$

- $\mathbf{w} = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{z} - c \cdot \mathsf{vk}$
- Assert $c = H(\mathbf{w}, \mathsf{msg})$
- Assert $\mathbf{z}$ short

In the ROM, the distribution of signatures of the above scheme is independent of the secret sk.

$\to$ allows to prove unforgeability

Threshold FSwA signature?

$\mathsf{FSwA.Sign(sk, msg)} \to \mathsf{sig}$

- $\mathbf{r} \leftarrow \chi_{\mathbf{r}}$
- $\mathbf{w} = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}$
- $c = H(\mathbf{w}, \mathsf{msg})$
- $\mathbf{z} = \mathsf{Rej}(c \cdot \mathsf{sk}, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M; \mathbf{r})$
- If $\mathbf{z} = \perp$ then **restart**
- Return $(c, \mathbf{z})$

Intuition N-out-of-N setting: $\mathsf{sk} = \sum_i \mathsf{sk}_i$

$\mathsf{TH\text{-}FSwA.Sign(sk, msg)} \to \mathsf{sig}$

Round 1:

- Sample a short $\mathbf{r}_i$
- $\mathbf{w}_i = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}_i$
- Broadcast $\mathsf{cmt}_i = H_{\mathsf{cmt}}(\mathbf{w}_i)$

Round 2:

- Broadcast $\mathbf{w}_i$

Round 3:

- $\mathbf{w} = \sum_i \mathbf{w}_i$
- $c = H(\mathbf{w}, \mathsf{msg})$
- Broadcast $\mathbf{z}_i = \mathsf{Rej}(c \cdot \mathsf{sk}_i, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M; \mathbf{r}_i)$

Combine: the final signature is

$$(c, \textstyle\sum_{i \in S} \mathbf{z}_i)$$

Threshold FSwA signature?

$\mathsf{FSwA.Sign}(\mathsf{sk}, \mathsf{msg}) \to \mathsf{sig}$

- $\mathbf{r} \leftarrow \chi_{\mathbf{r}}$
- $\mathbf{w} = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}$
- $c = H(\mathbf{w}, \mathsf{msg})$
- $\mathbf{z} = \mathsf{Rej}(c \cdot \mathsf{sk}, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M; \mathbf{r})$
- If $\mathbf{z} = \perp$ then **restart**
- Return $(c, \mathbf{z})$

- $\mathbf{w}_i$ is revealed even in case of rejection
  - Need proof strategy to show independence from secret
  - [DOTT22] hides rejected $\mathbf{w}_i$ with a trapdoor commitment scheme
  - [BTT22] simulates rejected $\mathbf{w}_i$ but with regularity lemma (degraded parameters)

Intuition N-out-of-N setting: $\mathsf{sk} = \sum_i \mathsf{sk}_i$

$\mathsf{TH\text{-}FSwA.Sign}(\mathsf{sk}, \mathsf{msg}) \to \mathsf{sig}$

Round 1:

- Sample a short $\mathbf{r}_i$
- $\mathbf{w}_i = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}_i$
- Broadcast $\mathsf{cmt}_i = H_{\mathsf{cmt}}(\mathbf{w}_i)$

Round 2:

- Broadcast $\mathbf{w}_i$

Round 3:

- $\mathbf{w} = \sum_i \mathbf{w}_i$
- $c = H(\mathbf{w}, \mathsf{msg})$
- Broadcast $\mathbf{z}_i = \mathsf{Rej}(c \cdot \mathsf{sk}_i, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M; \mathbf{r}_i)$

Combine: the final signature is

$$(c, \textstyle\sum_{i \in S} \mathbf{z}_i)$$

Threshold FSwA signature?

FSwA . Sign(sk, msg) → sig

- $\mathbf{r} \leftarrow \chi_{\mathbf{r}}$
- $\mathbf{w} = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}$
- $c = H(\mathbf{w}, \mathsf{msg})$
- $\mathbf{z} = \mathsf{Rej}(c \cdot \mathsf{sk}, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M; \mathbf{r})$
- If $\mathbf{z} = \perp$ then **restart**
- Return $(c, \mathbf{z})$

Intuition N-out-of-N setting: $\mathsf{sk} = \sum_i \mathsf{sk}_i$

TH-FSwA . Sign(sk, msg) → sig

Round 1:

- Sample a short $\mathbf{r}_i$
- $\mathbf{w}_i = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}_i$
- Broadcast $\mathsf{cmt}_i = H_{\mathsf{cmt}}(\mathbf{w}_i)$

Round 2:

- Broadcast $\mathbf{w}_i$

Round 3:

- $\mathbf{w} = \sum_i \mathbf{w}_i$
- $c = H(\mathbf{w}, \mathsf{msg})$
- Broadcast $\mathbf{z}_i = \mathsf{Rej}(c \cdot \mathsf{sk}_i, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M; \mathbf{r}_i)$

Combine: the final signature is

$$(c, \textstyle\sum_{i \in S} \mathbf{z}_i)$$

- $\mathbf{w}_i$ is revealed even in case of rejection
  - Need proof strategy to show independence from secret
  - [DOTT22] hides rejected $\mathbf{w}_i$ with a trapdoor commitment scheme
  - [BTT22] simulates rejected $\mathbf{w}_i$ but with regularity lemma (degraded parameters)

→ Tighter simulation lemma

Threshold FSwA signature?

Lemma: Rejected $\mathbf{w}_i$ is indistinguishable from uniform if:

- $\mathbf{w} = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}$ is indistinguishable from uniform, with $\mathbf{r} \leftarrow \chi_{\mathbf{r}}$
- $[\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{z}$ is indistinguishable from uniform, with $\mathbf{z} \leftarrow \chi_{\mathbf{z}}$

Threshold FSwA signature?

FSwA . Sign(sk, msg) $\rightarrow$ sig

- $\mathbf{r} \leftarrow \chi_{\mathbf{r}}$
- $\mathbf{w} = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}$
- $c = H(\mathbf{w}, \mathsf{msg})$
- $\mathbf{z} = \mathsf{Rej}(c \cdot \mathsf{sk}, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M; \mathbf{r})$
- If $\mathbf{z} = \perp$ then **restart**
- Return $(c, \mathbf{z})$

TH-FSwA . Sign(sk, msg) $\rightarrow$ sig

Round 1:

- Sample a short $\mathbf{r}_i$
- $\mathbf{w}_i = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}_i$
- Broadcast $\mathsf{cmt}_i = H_{\mathsf{cmt}}(\mathbf{w}_i)$

Round 2:

- Broadcast $\mathbf{w}_i$

Round 3:

- $\mathbf{w} = \sum_i \mathbf{w}_i$
- $c = H(\mathbf{w}, \mathsf{msg})$
- Broadcast $\mathbf{z}_i = \mathsf{Rej}(c \cdot \mathsf{sk}_i, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M; \mathbf{r}_i)$

Combine: the final signature is

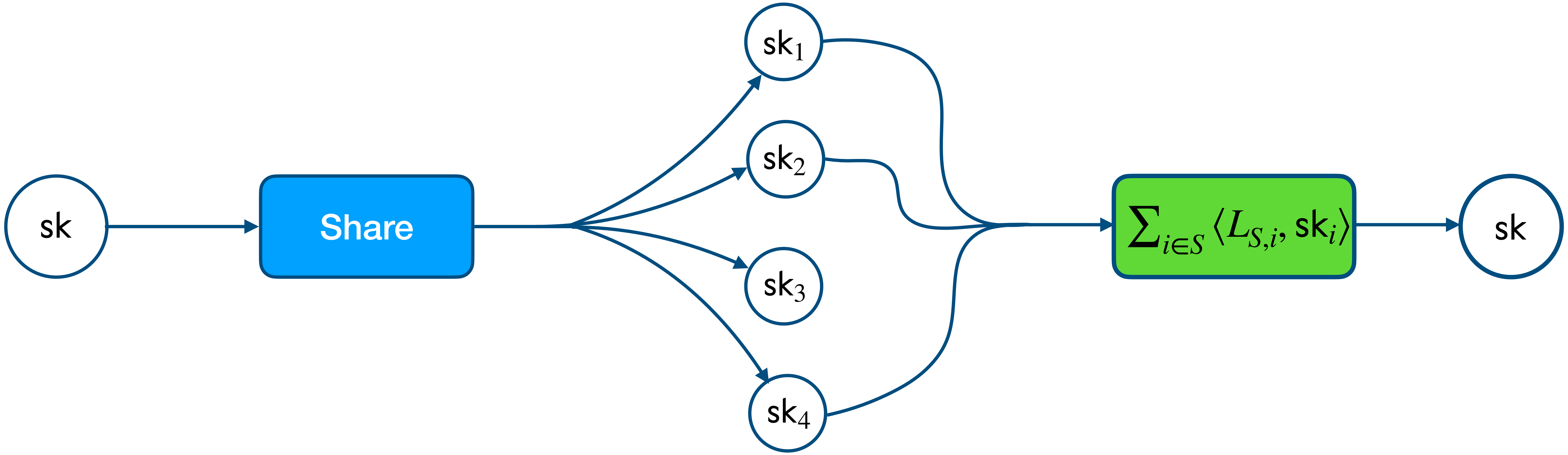
$$(c, \sum_{i \in S} \mathbf{z}_i)$$

- $\mathbf{w}_i$ is revealed even in case of rejection
  - Need proof strategy to show independence from secret
  - [DOTT22] hides rejected $\mathbf{w}_i$ with a trapdoor commitment scheme
  - [BTT22] simulates rejected $\mathbf{w}_i$ but with regularity lemma (degraded parameters)

$\rightarrow$ Tighter simulation lemma

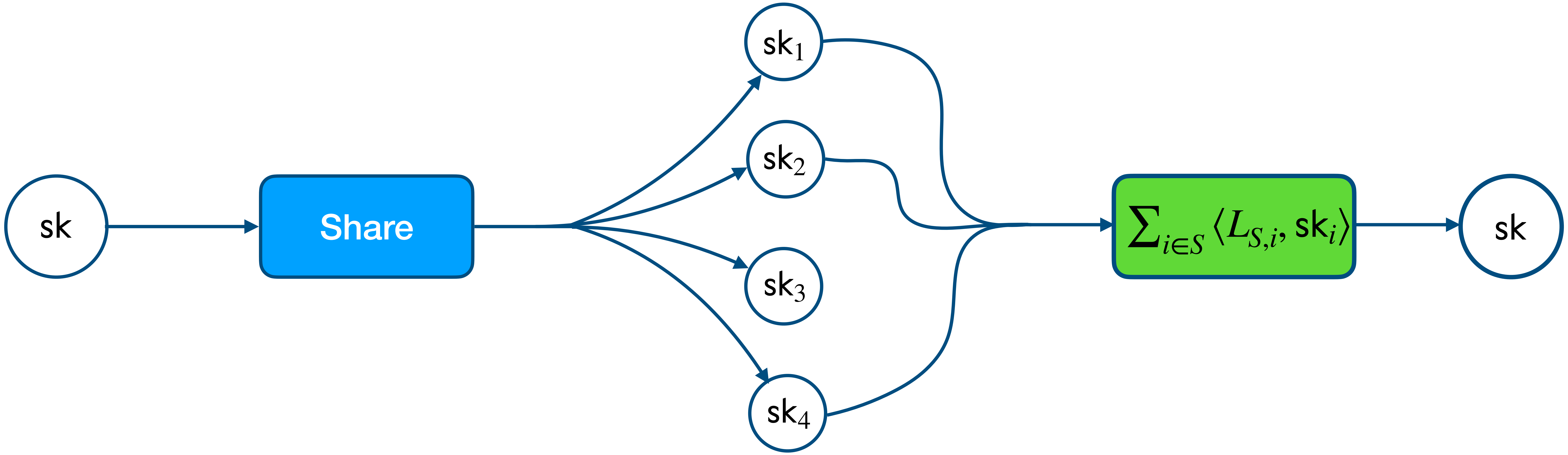
- How to support T-out-of-N?

Short secret sharing



- Individual pool of short shares $\mathsf{sk}_i = (\mathbf{s}_i^{(1)}, \mathbf{s}_i^{(2)}, \ldots)$
- T shares: can recover sk
  - Reconstruction vector $L_{S,i}$ with small coefficients
- $\leq T - 1$ shares: can't recover sk

Short secret sharing



- Individual pool of short shares $\mathsf{sk}_i = (\mathbf{s}_i^{(1)}, \mathbf{s}_i^{(2)}, \ldots)$
- T shares: can recover sk
  - Reconstruction vector $L_{S,i}$ with small coefficients
- $\leq T-1$ shares: can't recover sk

Example: N-out-of-N sharing (one share per party)

- $\mathsf{sk}_1, \ldots, \mathsf{sk}_N \leftarrow \mathcal{D}_\sigma^N$ and $\mathsf{sk} = \sum_i \mathsf{sk}_i$
- $L_{S,i} = 1$

Extends to T-out-of-N by having several shares per party.

Threshold FSwA signature?

$\mathsf{FSwA}.\mathsf{Sign}(\mathsf{sk}, \mathsf{msg}) \to \mathsf{sig}$

- $\mathbf{r} \leftarrow \chi_{\mathbf{r}}$
- $\mathbf{w} = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}$
- $c = H(\mathbf{w}, \mathsf{msg})$
- $\mathbf{z} = \mathsf{Rej}(c \cdot \mathsf{sk}, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M; \mathbf{r})$
- If $\mathbf{z} = \perp$ then **restart**
- Return $(c, \mathbf{z})$

- $\mathbf{w}_i$ is revealed even in case of rejection
  - Need proof strategy to show independence of secret
  - [DOTT22] hides rejected $\mathbf{w}_i$ with a trapdoor commitment scheme
  - [BTT22] simulates rejected $\mathbf{w}_i$ but with regularity lemma (degraded parameters)

  → Tighter simulation lemma

- How to support T-out-of-N?

  → Use short secret sharing

$\mathsf{TH\text{-}FSwA}.\mathsf{Sign}(\mathsf{sk}, \mathsf{msg}) \to \mathsf{sig}$

Round 1:

- Sample a short $\mathbf{r}_i$
- $\mathbf{w}_i = [\mathbf{A} \quad \mathbf{I}] \cdot \mathbf{r}_i$
- Broadcast $\mathsf{cmt}_i = H_{\mathsf{cmt}}(\mathbf{w}_i)$

Round 2:

- Broadcast $\mathbf{w}_i$

Round 3:

- $\mathbf{w} = \sum_i \mathbf{w}_i$
- $c = H(\mathbf{w}, \mathsf{msg})$
- Broadcast $\mathbf{z}_i = \mathsf{Rej}(c \cdot \langle L_{S,i}, \mathsf{sk}_i \rangle, \chi_{\mathbf{r}}, \chi_{\mathbf{z}}, M; \mathbf{r}_i)$

Combine: the final signature is

$$(c, \textstyle\sum_{i \in S} \mathbf{z}_i)$$

3. T-out-of-N short secret sharing

How to Shortly Share a Short Vector

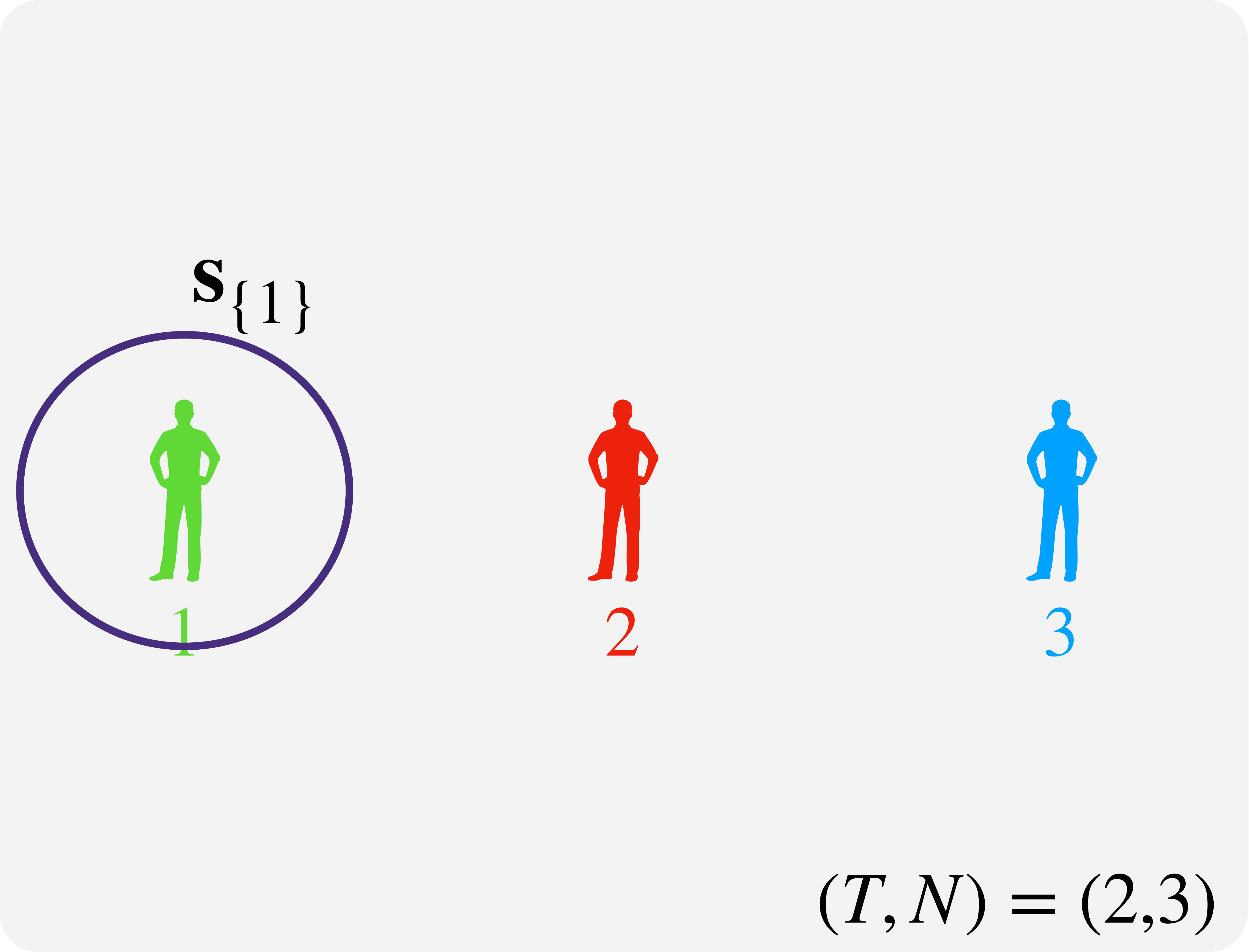
DKG with Short Shares and Application to Lattice-Based Threshold Signatures with Identifiable Aborts

Rafael del Pino[1], Thomas Espitau[1], Guilhem Niot[1,2], and Thomas Prest[1]

Solution: Replicated Secret Sharing

Idea: sample a share for any possible set of corrupted parties.

1. For any set $\mathscr{T}$ of $T-1$ parties, sample a uniform share $\mathbf{s}_{\mathscr{T}}$.

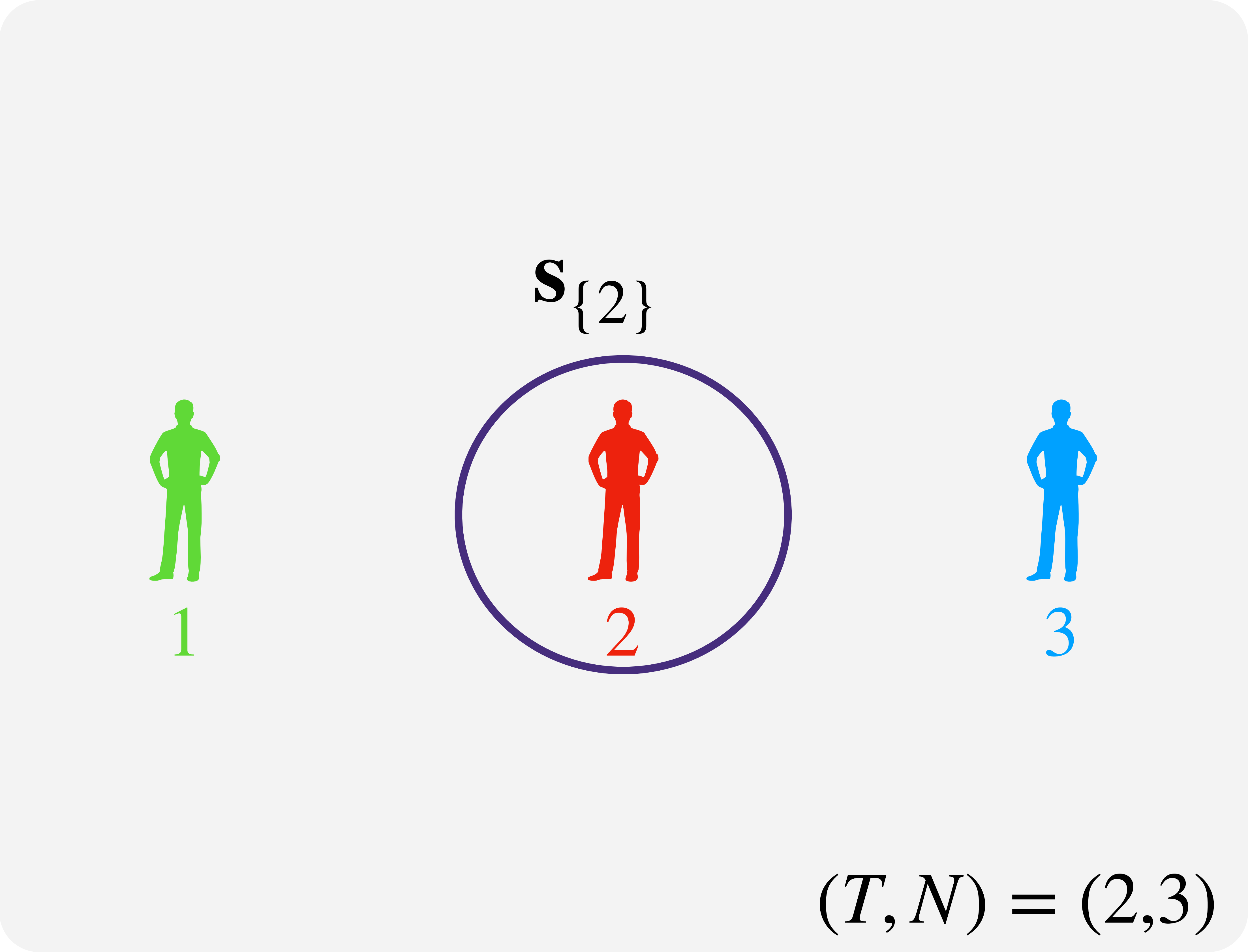


Solution: Replicated Secret Sharing

Idea: sample a share for any possible set of corrupted parties.

1. For any set $\mathcal{T}$ of $T-1$ parties, sample a uniform share $\mathbf{s}_{\mathcal{T}}$.

$\mathbf{s}_{\{1\}}$

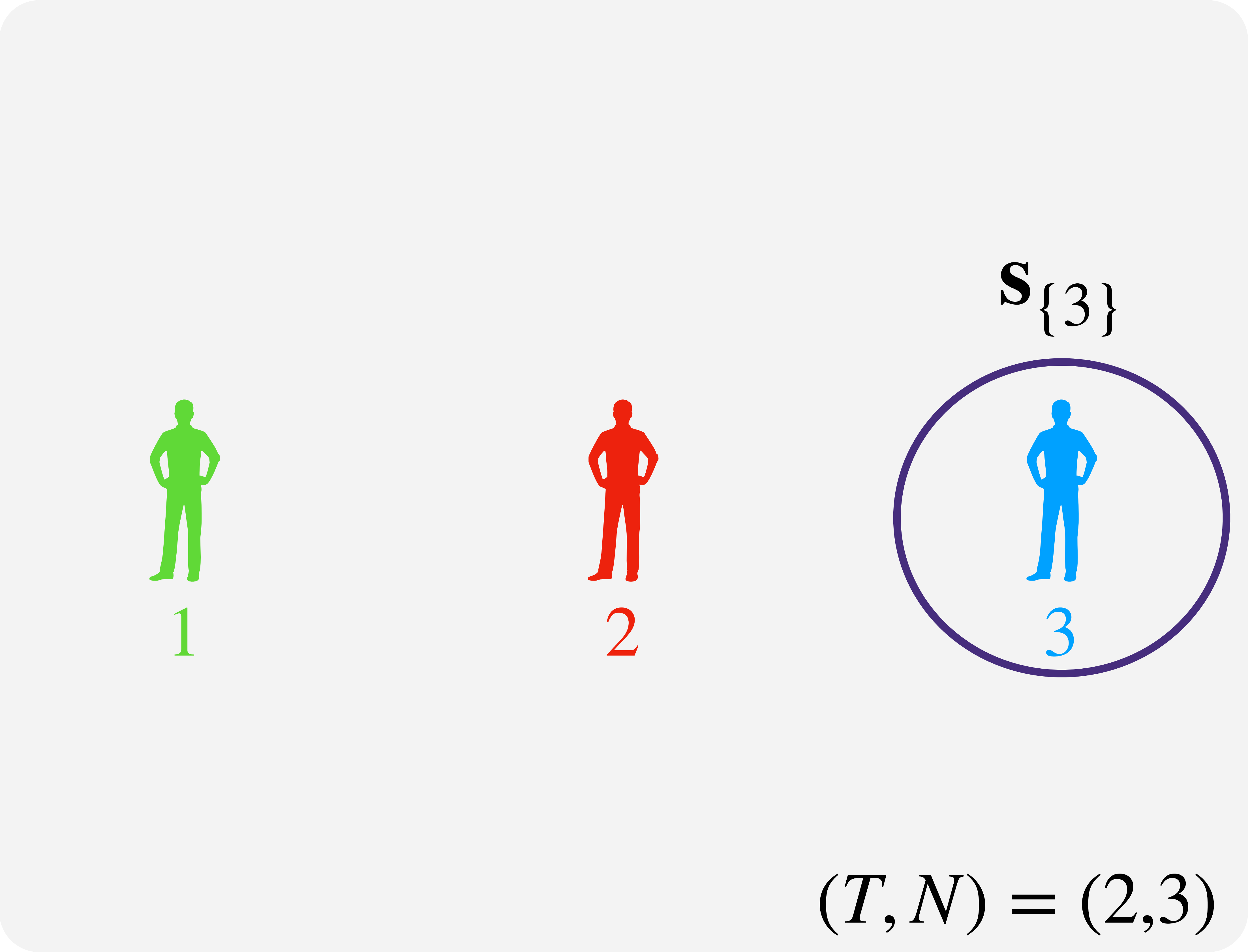


Solution: Replicated Secret Sharing

Idea: sample a share for any possible set of corrupted parties.

1. For any set $\mathscr{T}$ of $T-1$ parties, sample a uniform share $\mathbf{s}_{\mathscr{T}}$.

$\mathbf{s}_{\{1\}}$ $\mathbf{s}_{\{2\}}$

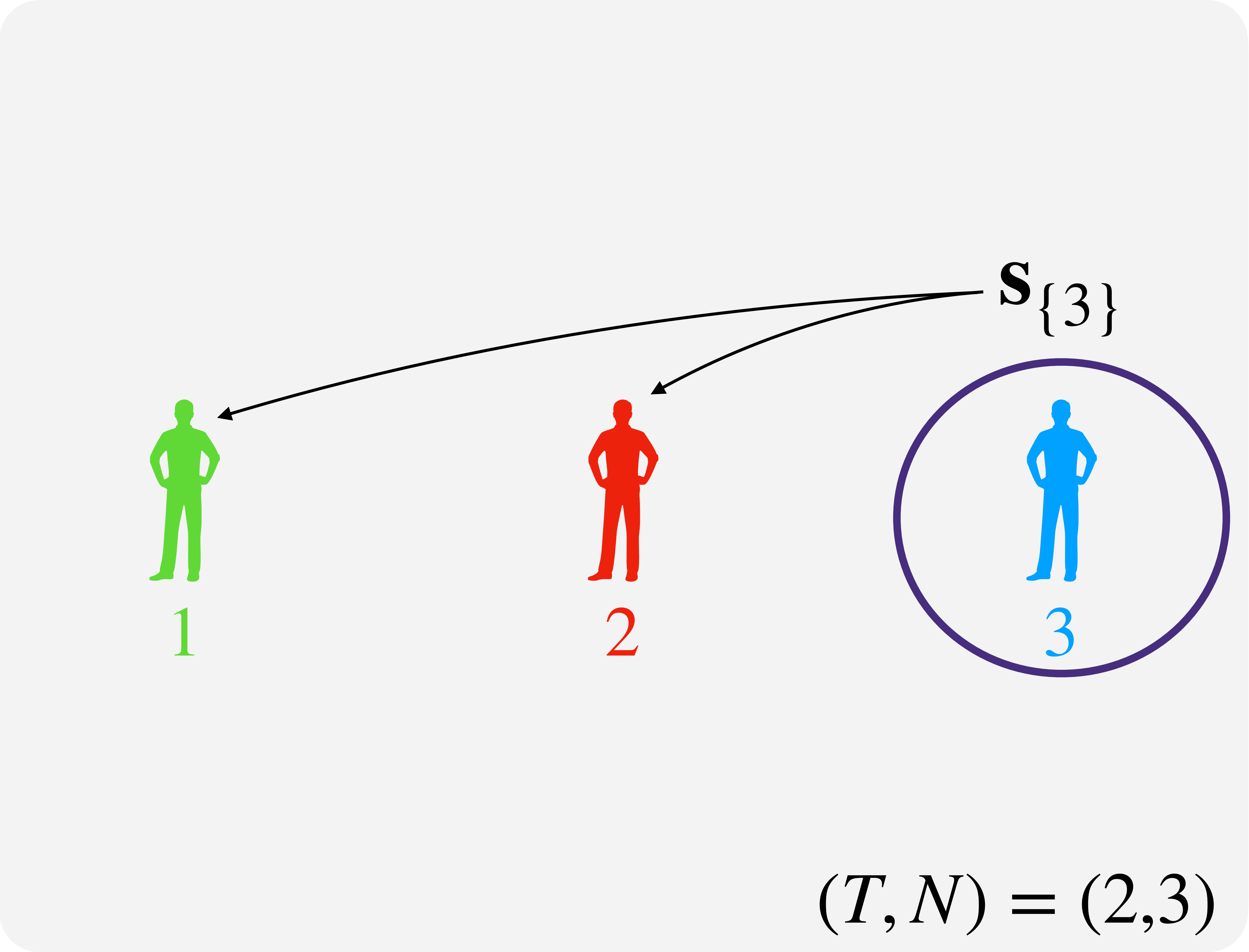


Solution: Replicated Secret Sharing

Idea: sample a share for any possible set of corrupted parties.

1. For any set $\mathcal{T}$ of $T-1$ parties, sample a uniform share $\mathbf{s}_{\mathcal{T}}$.

2. Distribute $\mathbf{s}_{\mathcal{T}}$ to the parties in $[N]\backslash\mathcal{T}$.



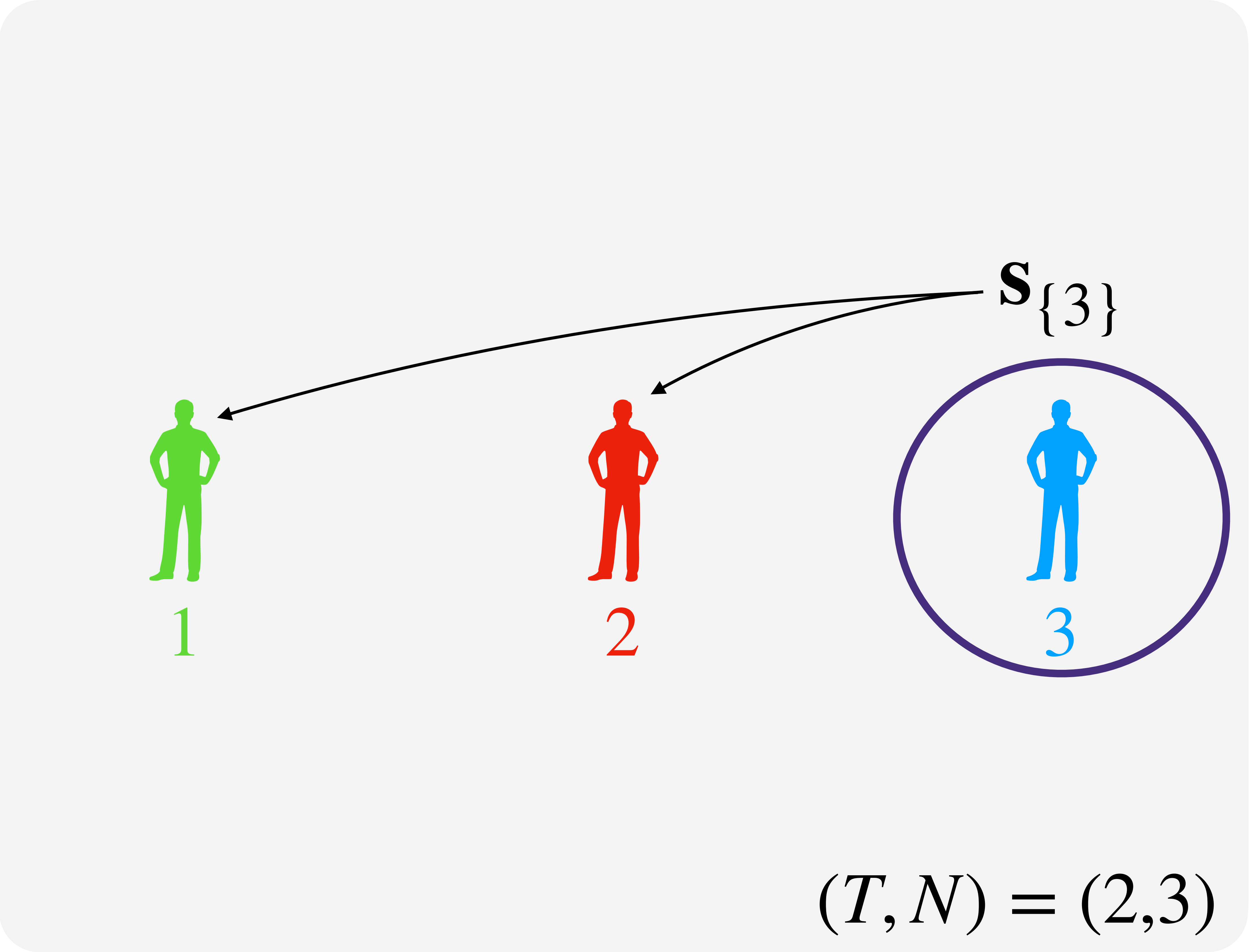
Solution: Replicated Secret Sharing

Idea: sample a share for any possible set of corrupted parties.

1. For any set $\mathcal{T}$ of $T-1$ parties, sample a uniform share $\mathbf{s}_{\mathcal{T}}$.

2. Distribute $\mathbf{s}_{\mathcal{T}}$ to the parties in $[N]\backslash\mathcal{T}$.

3. Define $\mathsf{sk} = \sum_{\mathcal{T}} \mathbf{s}_{\mathcal{T}}$.



Solution: Replicated Secret Sharing

Idea: sample a share for any possible set of corrupted parties.

1. For any set $\mathcal{T}$ of $T-1$ parties, sample a uniform share $\mathbf{s}_{\mathcal{T}}$.
2. Distribute $\mathbf{s}_{\mathcal{T}}$ to the parties in $[N] \backslash \mathcal{T}$.
3. Define $\mathsf{sk} = \sum_{\mathcal{T}} \mathbf{s}_{\mathcal{T}}$.

Properties:

- Reconstruction coefficients 0 or 1
- When $< T$ corrupted parties, at least one $\mathbf{s}_{\mathcal{T}}$ remains hidden.

$\rightarrow$ guarantees that sk remains protected

Solution: Short Replicated Secret Sharing

Idea: sample a share for any possible set of corrupted parties.

1. For any set $\mathscr{T}$ of $T-1$ parties, sample a short share $\mathbf{s}_{\mathscr{T}}$.

2. Distribute $\mathbf{s}_{\mathscr{T}}$ to the parties in $[N] \backslash \mathscr{T}$.

3. Define sk $= \sum_{\mathscr{T}} \mathbf{s}_{\mathscr{T}}$.

Properties:

- Reconstruction coefficients 0 or 1
- When $< T$ corrupted parties, at least one $\mathbf{s}_{\mathscr{T}}$ remains hidden.

  $\rightarrow$ guarantees that $[\mathbf{A} \quad \mathbf{I}] \cdot$ sk looks uniform (MLWE assumption)

Solution: Short Replicated Secret Sharing

Idea: sample a share for any possible set of corrupted parties.

1. For any set $\mathscr{T}$ sample a short

2. Distribute $\mathbf{s}_{\mathscr{T}}$ t $[N]\backslash\mathscr{T}$.

3. Define sk $= \sum_{\mathscr{T}} \mathbf{s}_{\mathscr{T}}$.

Caveat: This scheme has a number of shares that is equal to $\binom{N}{T-1}$.

efficients 0 or 1

ted parties, at least one $\mathbf{s}_{\mathscr{T}}$ remains hidden.

→ guarantees that $[\mathbf{A} \quad \mathbf{I}] \cdot$ sk looks uniform (MLWE assumption)

Threshold FSwA signature

For $N \leq 8$,

Distributions	**Speed**	**Rounds**	**| vk |**	**| sig |**	**Total communication**
Gaussians	Fast	3	2.6 kB	2.7 kB	5.6 kB
Uniforms			3.1 kB	4.8 kB	13.5 kB

Comparable to Dilithium size: 2.4kB at NIST level II!

Conclusion

Conclusion

- **Introduced Finally, a 3-round compact lattice-based threshold signature**
  - Up to 8 parties
  - Signature size 2.7kB (comparable to Dilithium, 2.4kB)
- **Future work?**
  - Techniques applied to thresholdize ML-DSA: up to 5 parties
  - 2-round?
  - Tackle malicious behaviour?

Questions?