Flood and Submerse:

Distributed Key Generation and Robust Threshold Signature from Lattices

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ENSL/CWI/KCL/IRISA Joint Seminar - 30 Sept. 2024

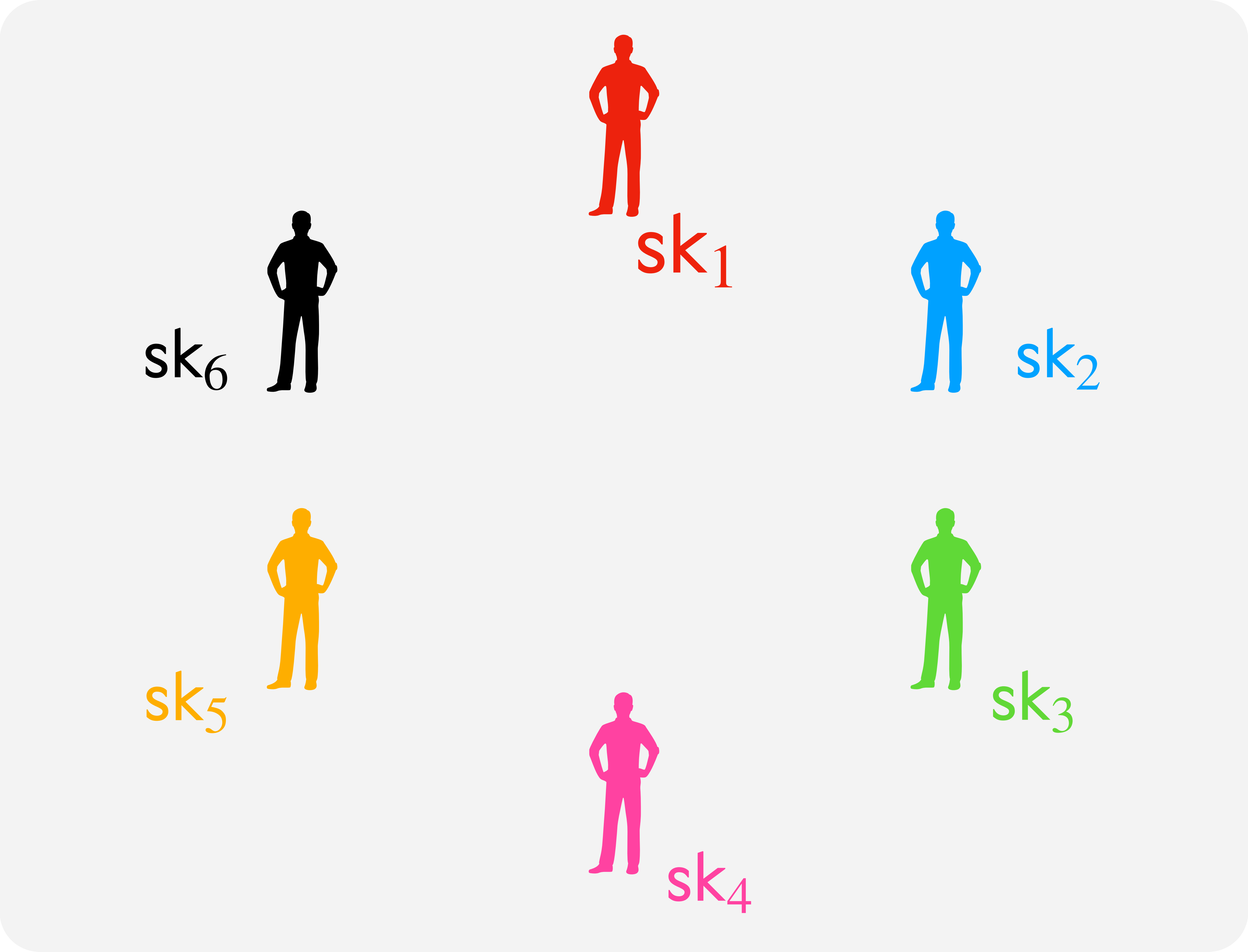
PQ SHIELD

1. Background

(T-out-of-N) threshold signatures

What are they?

An interactive protocol to distribute signature generation.



- 1 verification key vk
- 1 partial signing key sk_i per party
- Given at least T-out-of-N partial signing keys, we can sign.

(T-out-of-N) threshold signatures

What are they?

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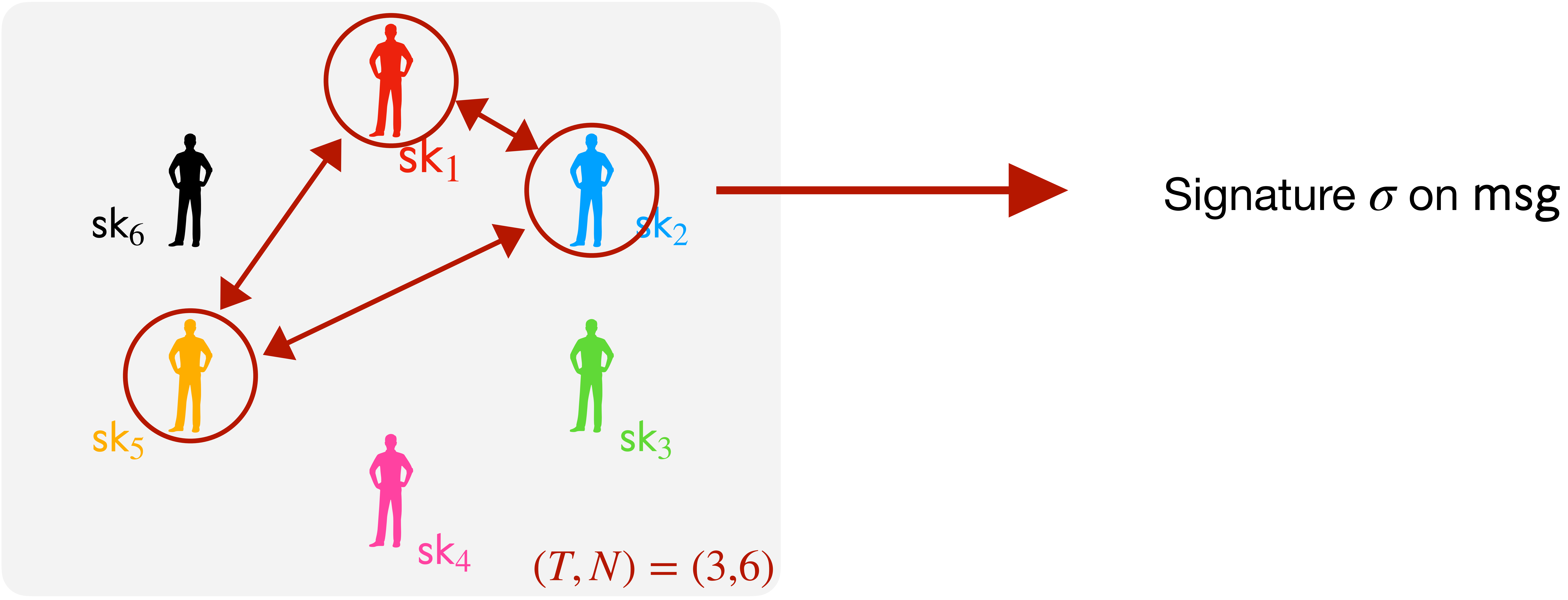
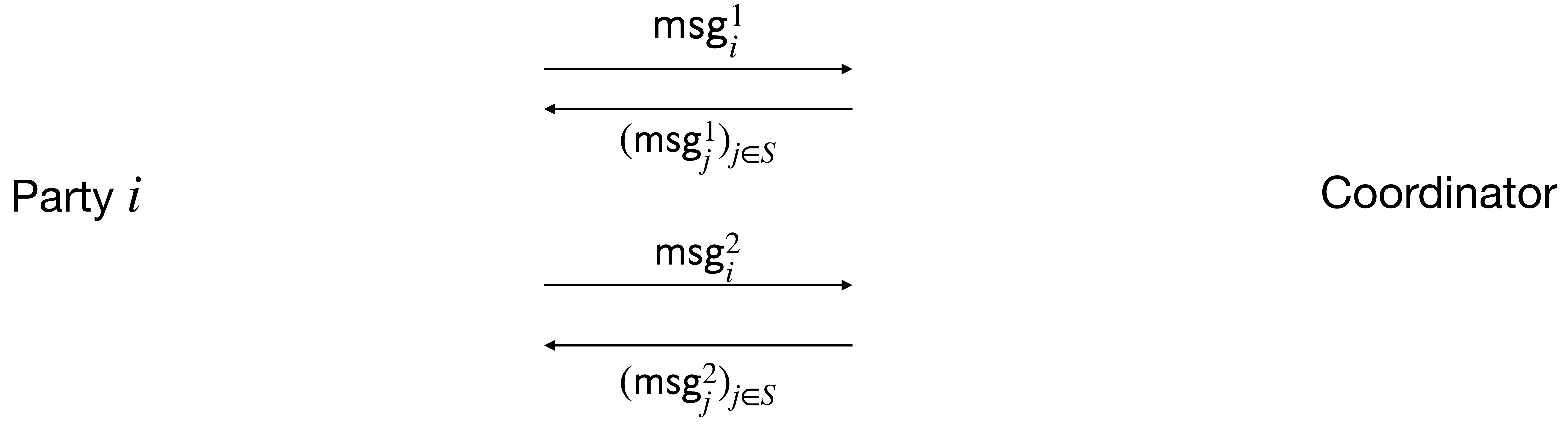


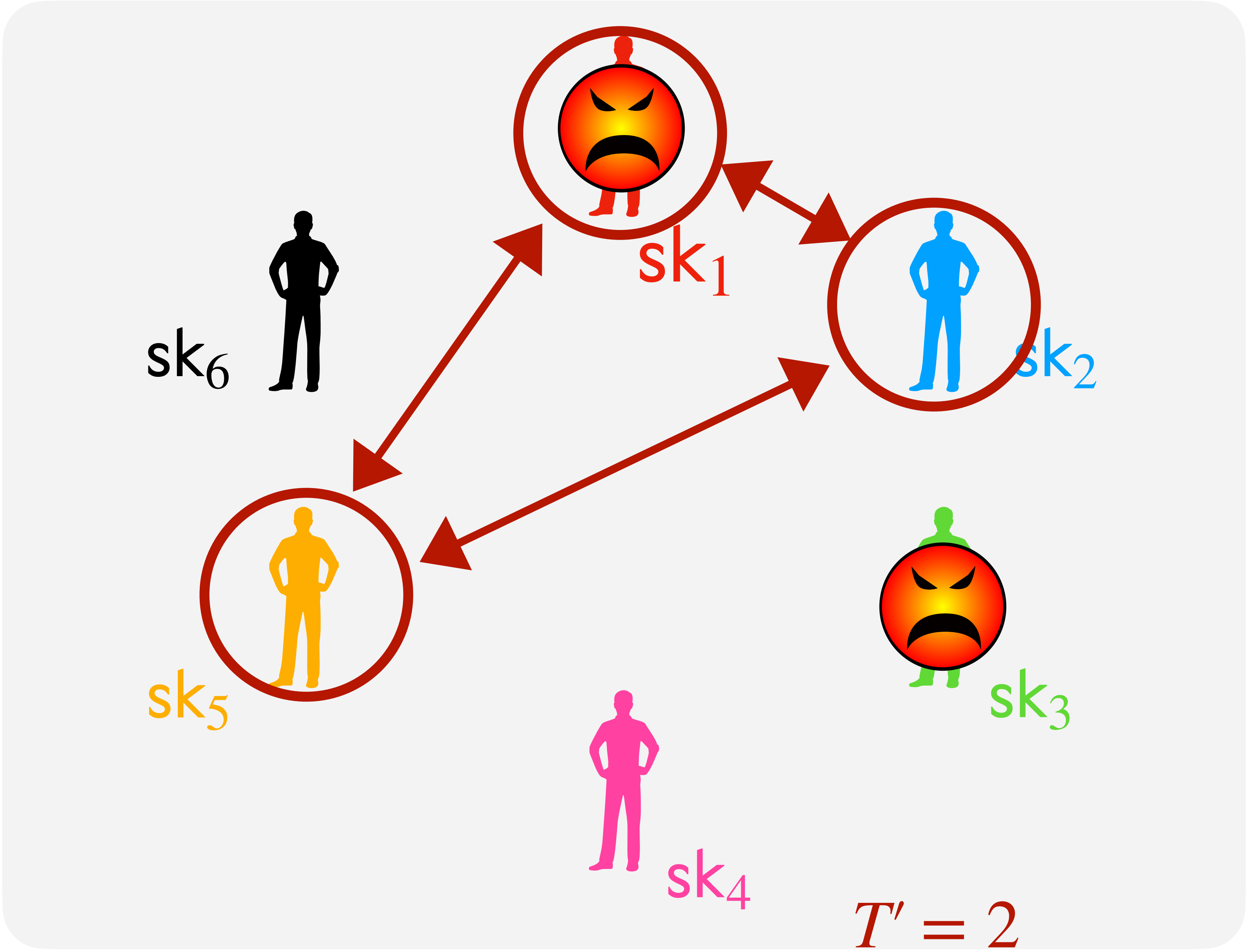
(T-out-of-N) threshold signatures
What are they?

Round-based communication model:



Core security properties

- **Correctness:** Given at least T-out-of-N partial signing keys, we can sign.
- **Unforgeability:** The signature scheme remains unforgeable even if up to $T' < T$ parties are corrupted. Often $T' = T - 1$.



It's not possible to forge a new signature, even by taking part in the signing protocol.

More desirable properties

- **Adaptive security:** (vs static security) Corrupted users can be chosen adaptively over the lifetime of the signature scheme. More realistic than static security, i.e. corrupted users chosen before setup.
- **Distributed Key Generation:** Protocol allowing to distributively sample key material.
- **Robustness (resp. identifiable abort):** In the presence of malicious users, signature protocol is guaranteed to produce a valid signature (resp. to identify misbehaving users)
- **Small round complexity:** Ideally can be as low as one round.
- **Backward compatibility:** Threshold schemes should ideally be compatible with existing primitives.

Pre-quantum solutions

- Mature solutions:
  - EdDSA: FROST [KG20]
  - ECDSA: [ANOS+21]
  - BLS: [Bol03]
  - RSA: [Sho00]

- Provide all desirable properties.

An active field of research for post-quantum security

- Aggregating hash-based signatures: [KCLM22]
- Sequential TS scheme based on isogenies: [DM20]
- Lattice-based threshold signatures:
  - 2-round TS via FHE: [BGG+18], [ASY22], [GKS23]
  - TS with noise flooding (based on Raccoon): 3-round [dPKM+23], 2-round [EKT24], [BKLM+24], 5-round adaptively secure [KRT24]

Threshold Raccoon: Practical Threshold Signatures from Standard Lattice Assumptions

Rafael del Pino[1], Shuichi Katsumata[1,2], Mary Maller[1,3], Fabrice Mouhartem[4], Thomas Prest[1], Markku-Juhani Saarinen[1,5]

Adaptively Secure 5 Round Threshold Signatures from MLWE/MSIS and DL with Rewinding

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Ringtail: Practical Two-Round Threshold Signatures from Learning with Errors

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Threshold Raccoon, a practical 3-round threshold signature

κ	Number Signers	| vk |	| sig |	Total communication
128	≤ 1024	4 kB	13 kB	40 kB

… but only considers core security properties: correctness and unforgeability.

Advanced properties of lattice-based schemes

Active research since 2024.

- **Adaptive security:** 5-round [KRT24]
- **Small round complexity:** 2-round [EKT24], [BKLM+24]
- **Backward compatibility:** These schemes can be made compatible with the NIST proposal Raccoon.

No efficient solution for:

- **Distributed Key Generation (DKG)**
- **Robustness / identifiable abort**

Focus of this presentation

- **Distributed Key Generation** (DKG)
- **Robustness**: Guarantee valid signature in the presence of malicious signers

Our techniques for DKG + robust signing are quite generic:

- in our paper, applied to Plover [EENP+24]: hash-and-sign scheme
- can be applied to all **3-round [dPKM+23]**, 2-round [EKT24], [BKLM+24]

Raccoon signature scheme

Lyubashevsky's signature scheme (without aborts)

$$\mathsf{vk} = \mathbf{t} = \underbrace{\begin{bmatrix}\mathbf{A}' & \mathbf{I}\end{bmatrix}}_{\mathbf{A}} \cdot \mathbf{s} \in \mathscr{R}_q^k \qquad \mathsf{sk} = \mathbf{s} \in \mathscr{R}_q^\ell \text{ short}$$

$\mathbf{r} \leftarrow \chi$

$\mathbf{w} = \mathbf{A} \cdot \mathbf{r}$ $\xrightarrow{\mathbf{w}}$

$\longleftarrow$ $c = H(\mathsf{vk}, \mathsf{msg}, \mathbf{w}) \in \mathscr{R}_q$ "small"

$\mathbf{z} = c \cdot \mathbf{s} + \mathbf{r}$ $\longrightarrow$ Accept if

- $\mathbf{z}$ is short
- $\mathbf{A} \cdot \mathbf{z} = c \cdot \mathbf{t} + \mathbf{w}$

Prove security via Hint-MLWE assumption

Hint-MLWE assumption [KLSS23]

Consider $\mathbf{t} = \underbrace{\begin{bmatrix} \mathbf{A}' & \mathbf{I} \end{bmatrix}}_{\mathbf{A}} \cdot \mathbf{s} \in \mathcal{R}_q^k$ and reveal hints $(\mathbf{z}_i = c_i \cdot \mathbf{s} + \mathbf{r}_i)_{i \in [Q]}$

t is indistinguishable from uniform (as hard as MLWE) for some parameter regimes.

Rule of thumb: secure if $\sigma_{\mathbf{r}} \approx \sqrt{Q} \cdot s_1(c) \cdot \sigma_{\mathbf{s}}$

Threshold Raccoon [dPKM+23]

Threshold signature: use (T, N)-Shamir sharing on secret

$\mathsf{sk} = \mathbf{s} \in \mathcal{R}_q^\ell$ short

Sample polynomial $f \in \mathcal{R}_q^\ell[X]$ s.t.

- $f(0) = \mathbf{s}$ and $\deg f = T - 1$
- Partial signing keys $\mathsf{sk}_i := [\![\mathbf{s}]\!]_i = f(i)$

For any set S of T shares, reconstruct $\mathbf{s}$:

$$\mathbf{s} = \sum_{i \in S} L_{S,i} \cdot [\![\mathbf{s}]\!]_i$$

Lagrange coefficient

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$[\![\mathbf{z}]\!]_i = c \cdot L_{S,i} \cdot [\![\mathbf{s}]\!]_i + \mathbf{r}_i + \Delta_i$ $\quad \mathbf{z}_i$ →

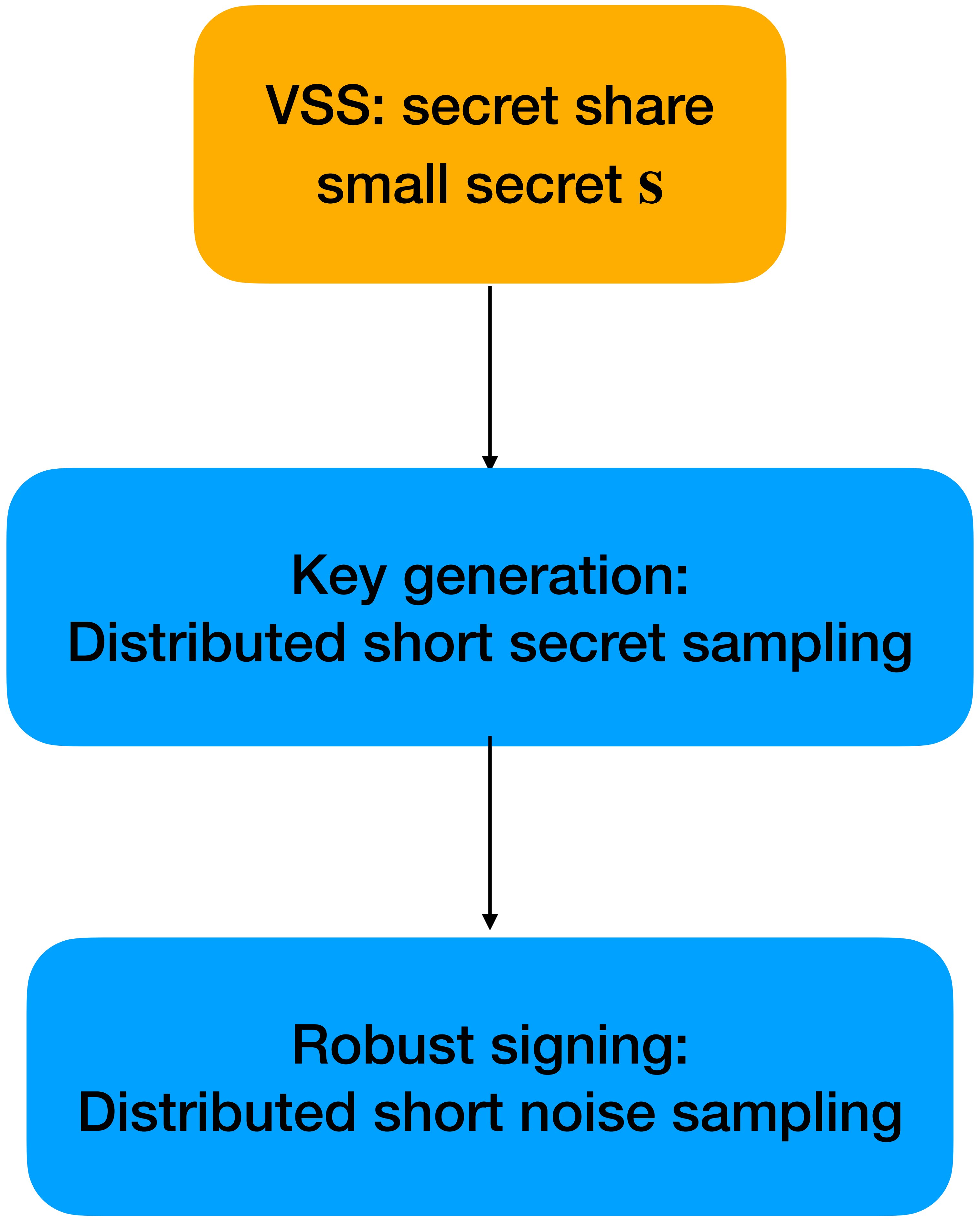
Additive sharing of 0

Accept if

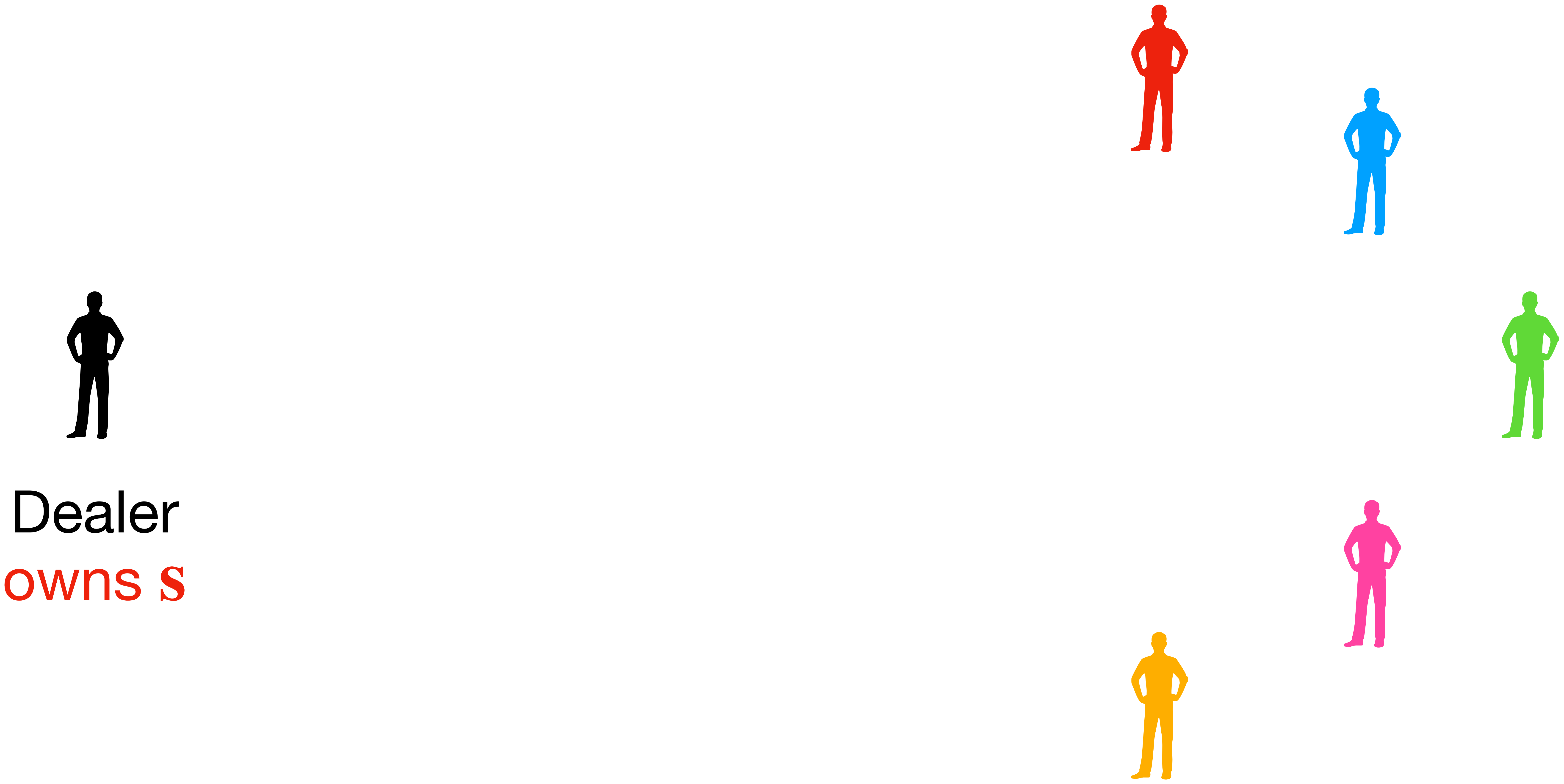
- $\mathbf{z} = \sum_{j \in S} [\![\mathbf{z}]\!]_j = c \cdot \mathbf{s} + \sum_{j \in S} \mathbf{r}_j$ is short
- $\mathbf{A} \cdot \mathbf{z} = c \cdot \mathbf{t} + \mathbf{w}$

2. Achieving additional threshold properties with Verifiable Secret Sharing

Achieving additional threshold properties with Verifiable Secret Sharing

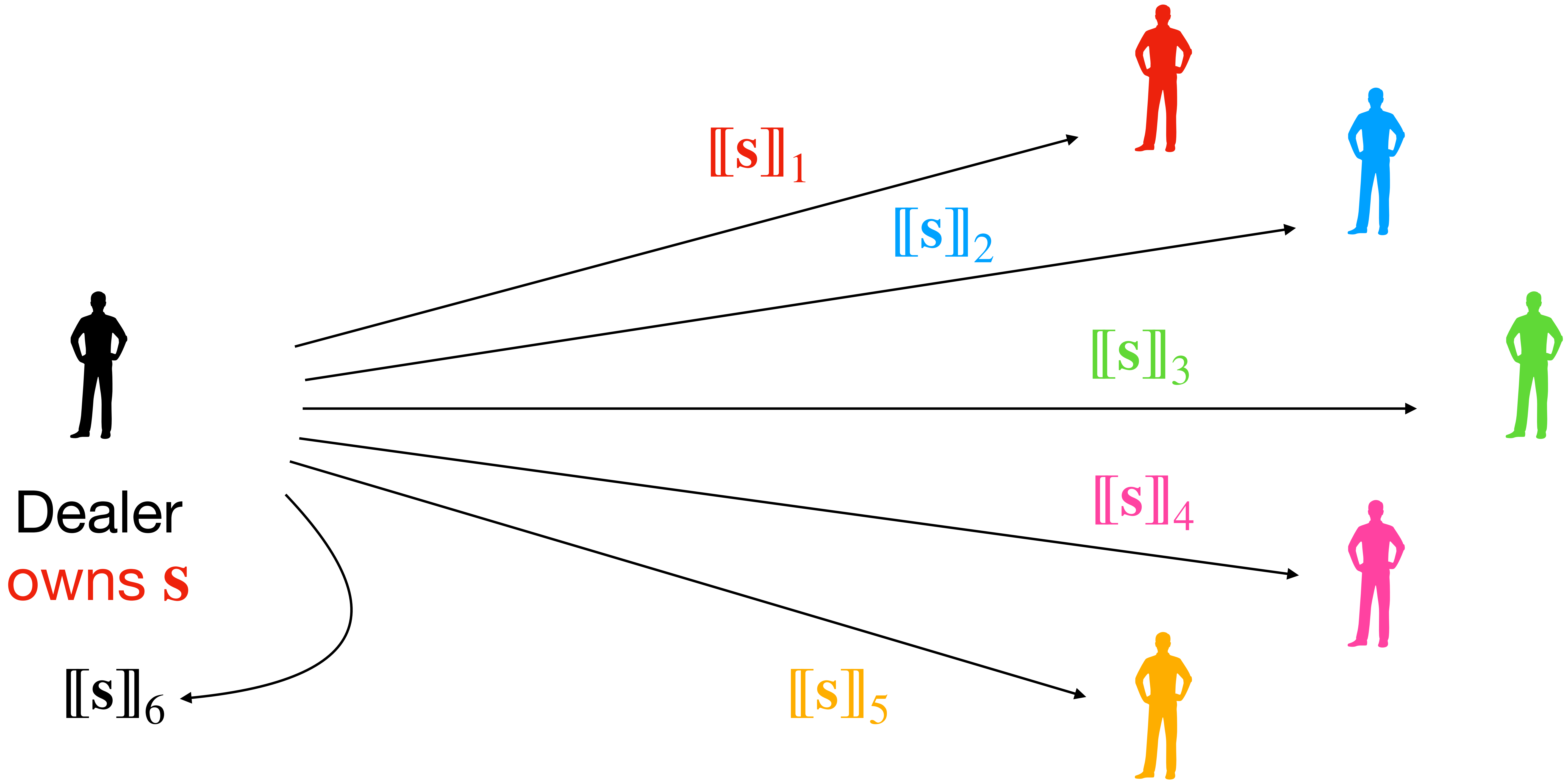


Verifiable Secret Sharing (VSS)



Verifiable Secret Sharing (VSS)

1) Send individual shares



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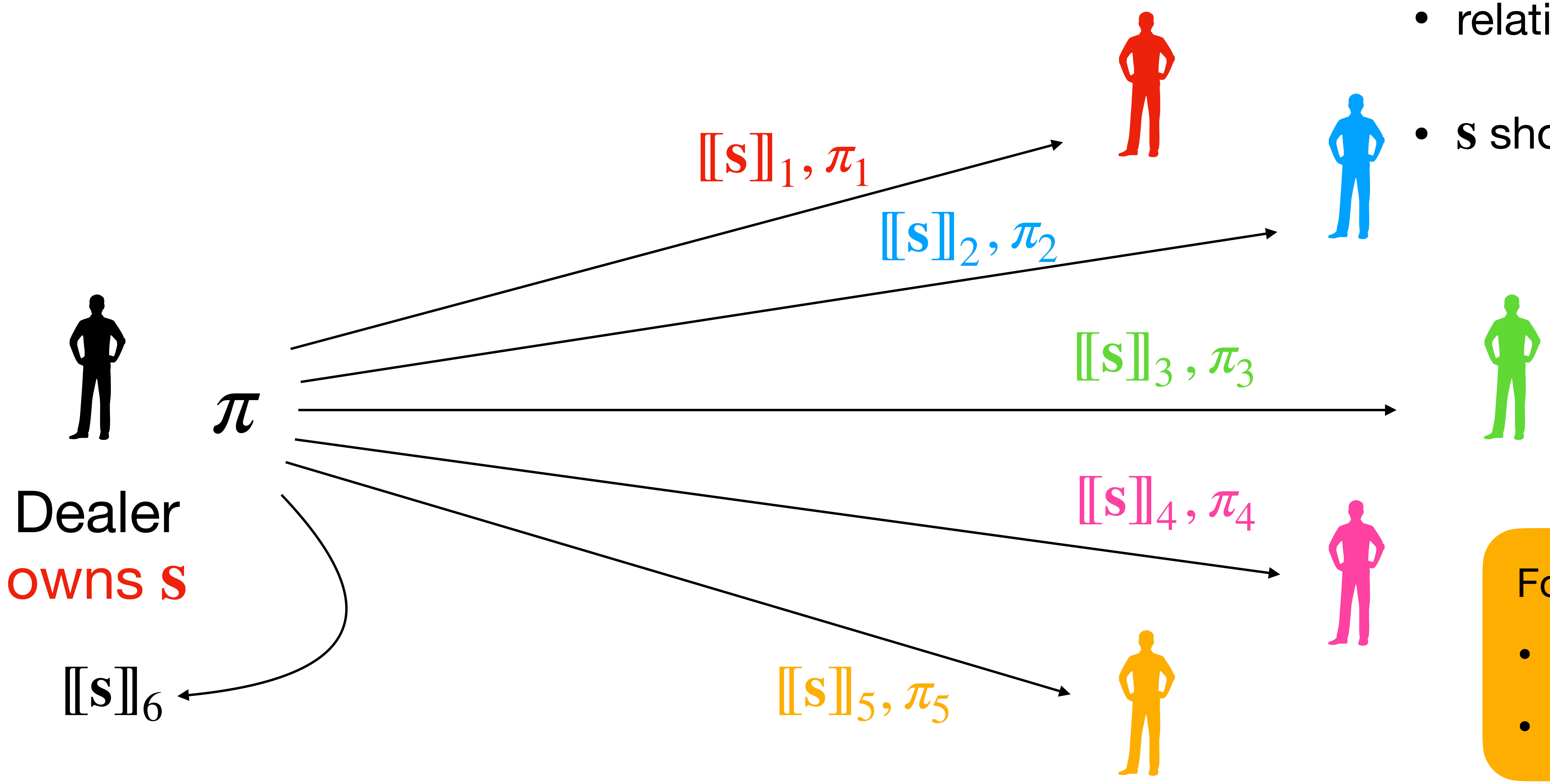


2) Prove correct sharing, i.e.

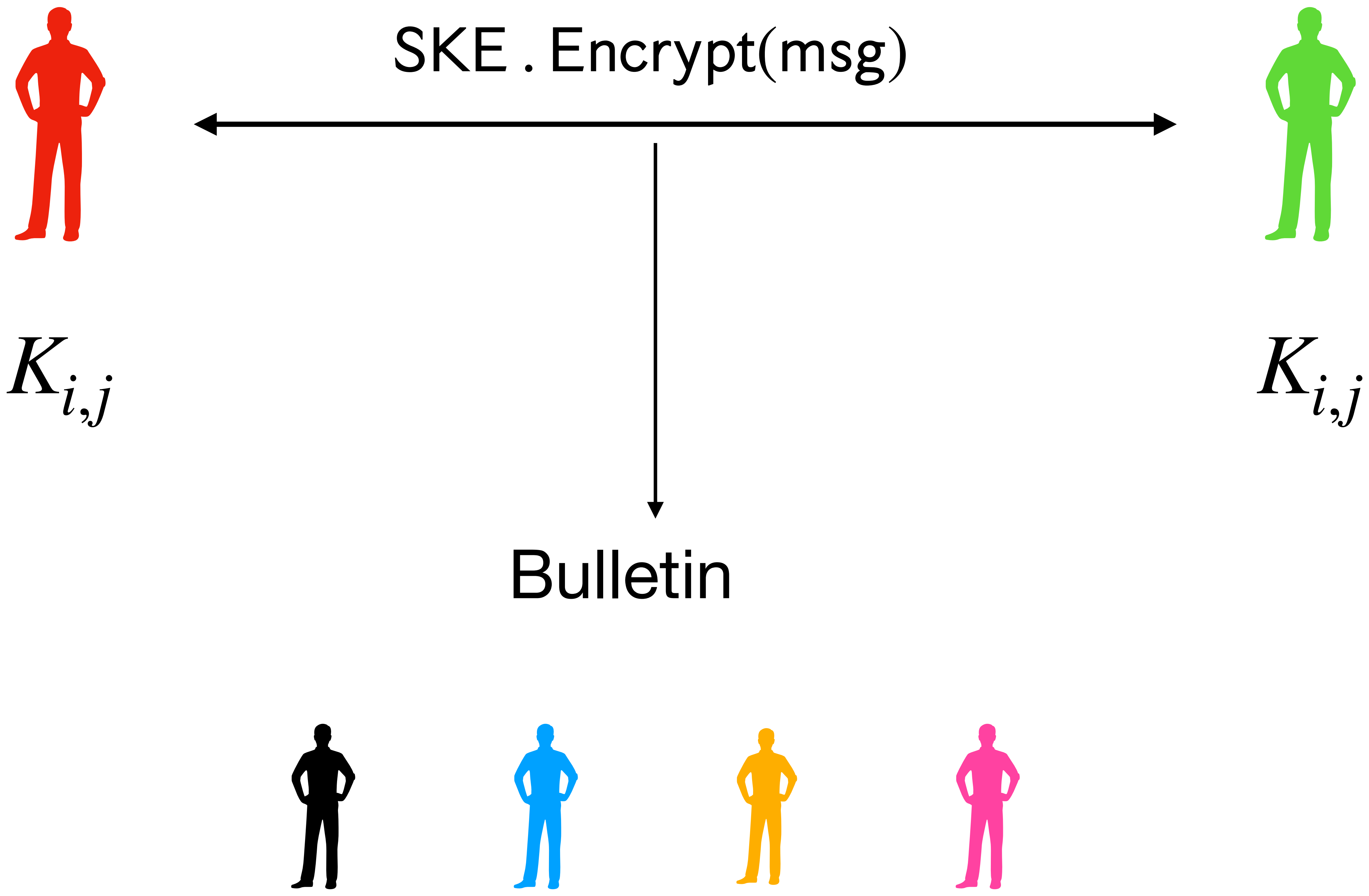
- relation $\mathbf{s} = \sum_{i \in S} L_{S,i} \cdot [\![\mathbf{s}]\!]_i$ for $|S| = T$
- $\mathbf{s}$ short

Formally,

- $\mathsf{VSS.Share}(\mathbf{s}) \rightarrow (\pi, ([\![\mathbf{s}]\!]_i, \pi_i)_{1 \leq i \leq N})$
- $\mathsf{VSS.Verify}(i, [\![\mathbf{s}]\!]_i, \pi, \pi_i) \rightarrow \mathbf{ok} \,|\, \mathbf{fail}$

Distributed Key Generation (DKG) from VSS

- Assume the existence of a broadcast or bulletin board.
- Assume the existence of non-repudiable pairwise channels.



Allows to prove that a message was sent.

Distributed Key Generation (DKG) from VSS

$$\text{vk} = \mathbf{t} = \underbrace{[\mathbf{A}' \mid \mathbf{I}]}_{\mathbf{A}} \cdot \mathbf{s} \in \mathscr{R}_q^k$$

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1. Construct and share secret key $\mathbf{s}$

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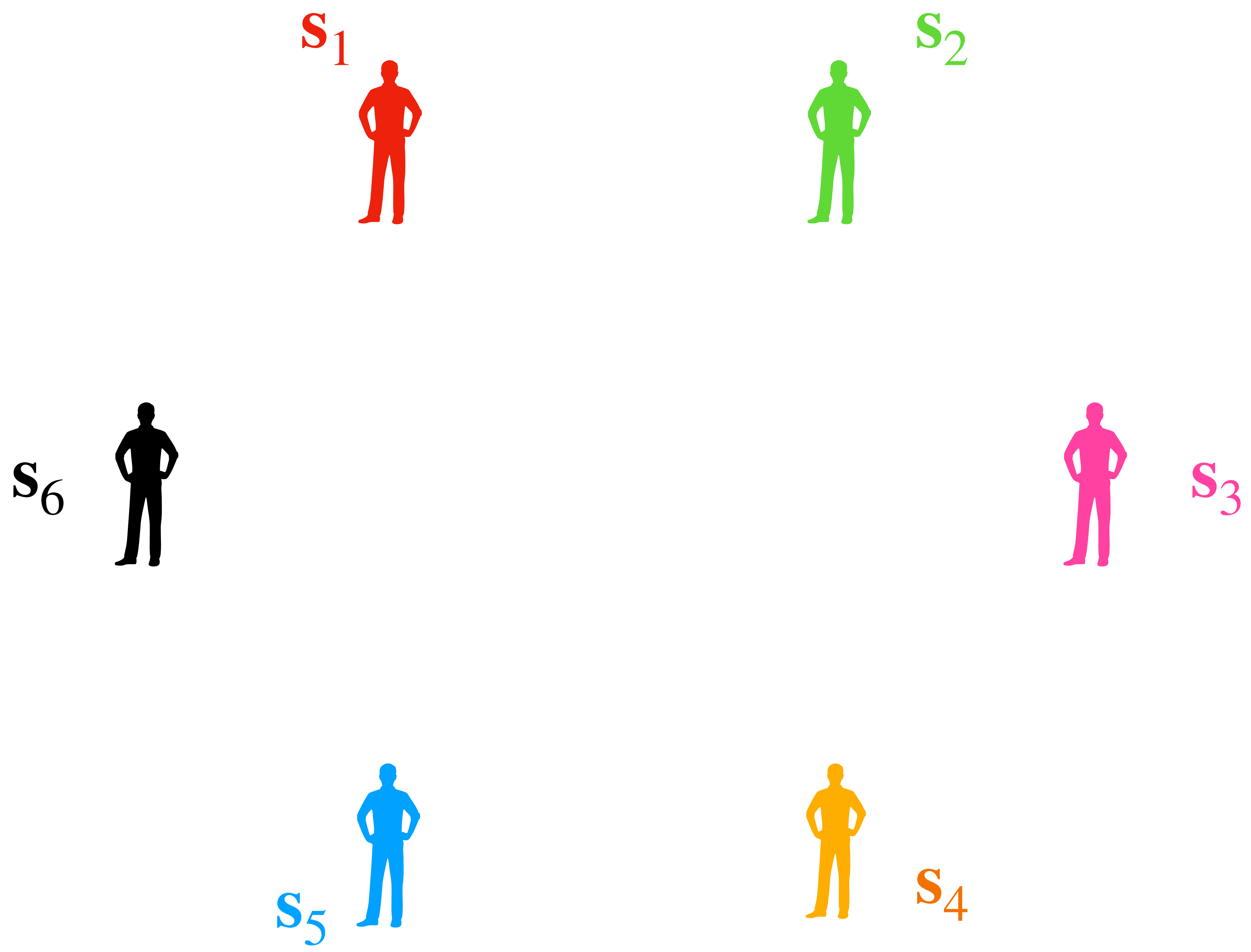
1. Construct and share secret key $\mathbf{s} = \sum_i \mathbf{s}_i$

1.a) Sample small secrets $\mathbf{s}_i$



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1. Construct and share secret key $\mathbf{s} = \sum_i \mathbf{s}_i$

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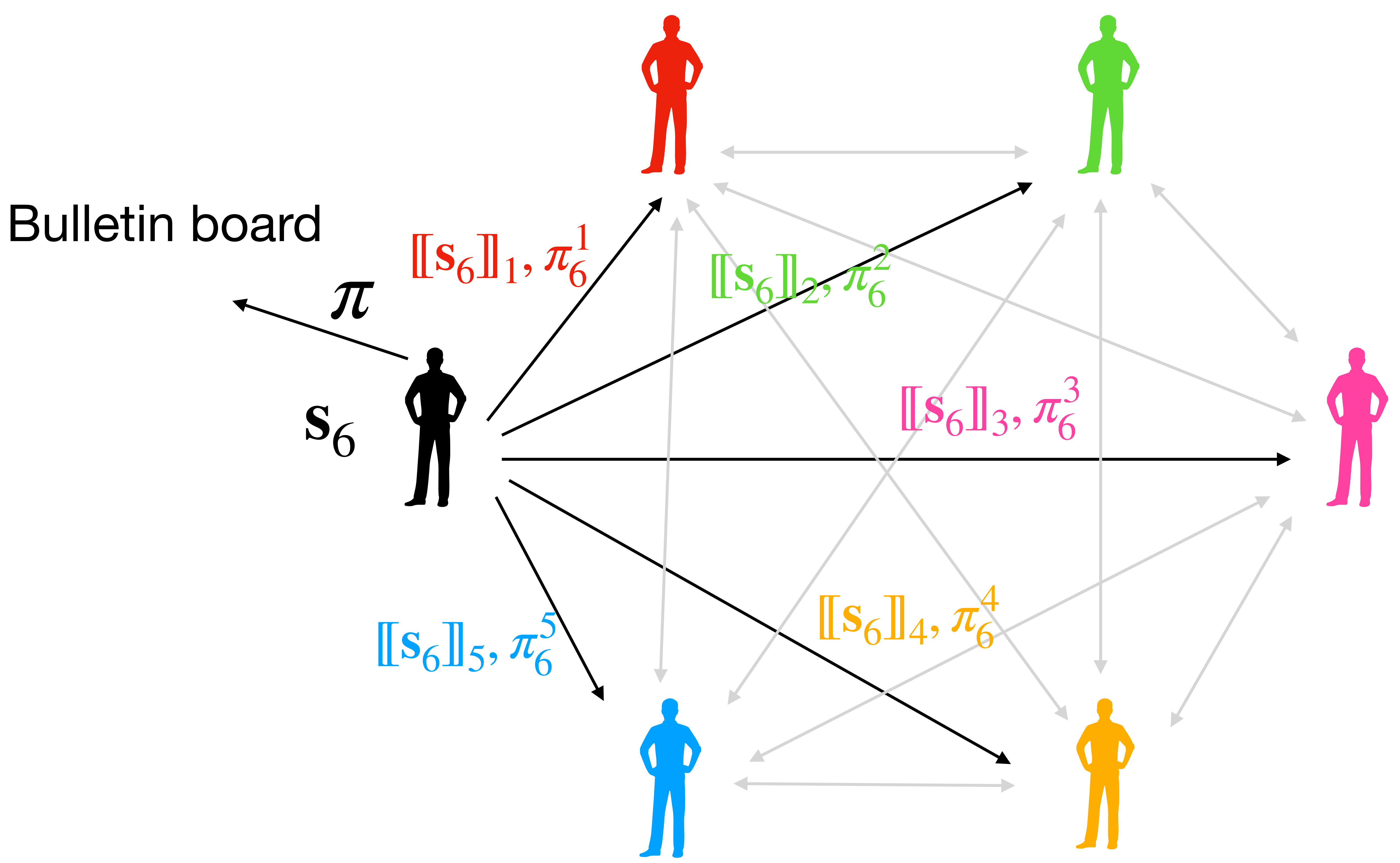

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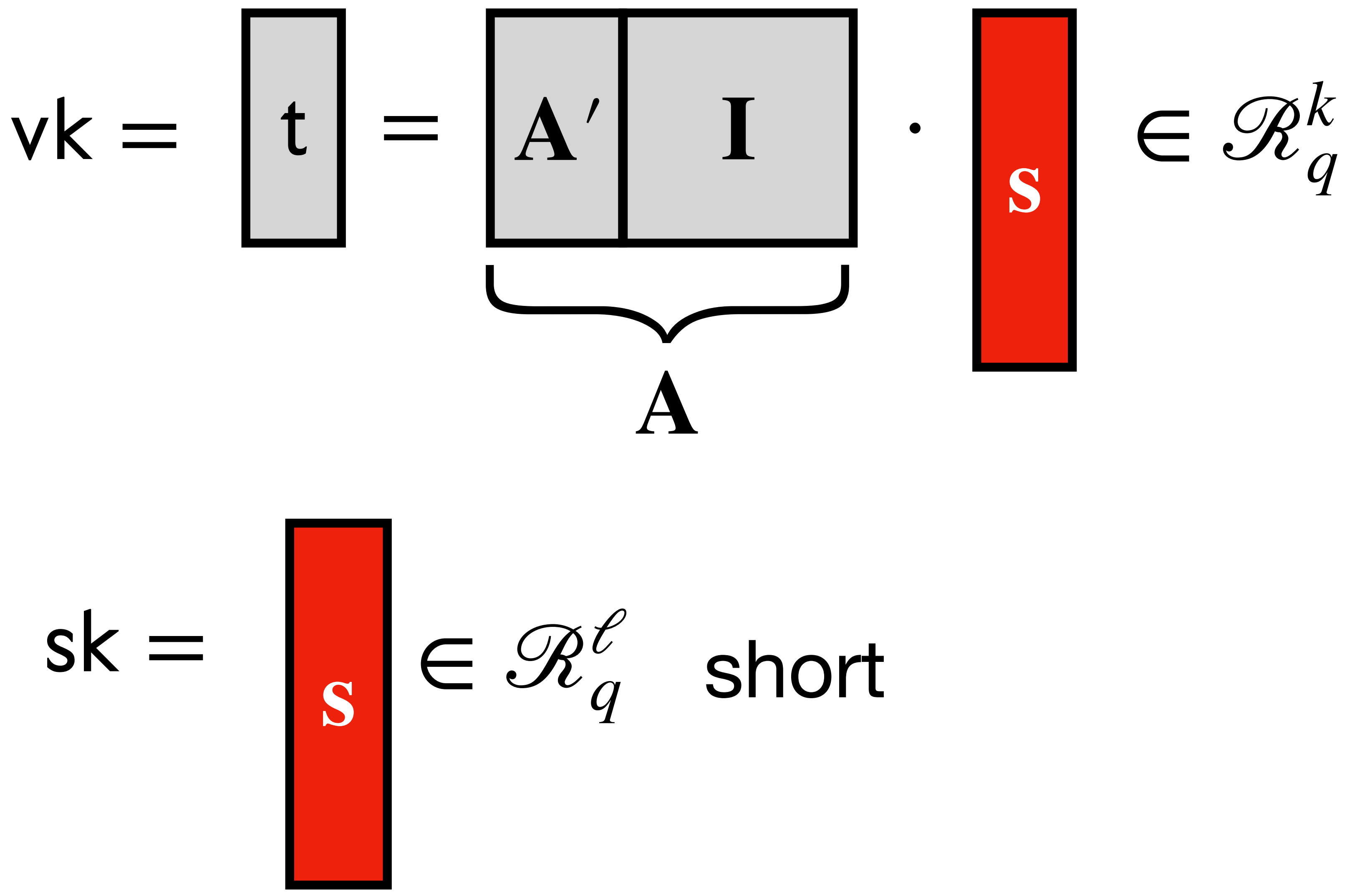

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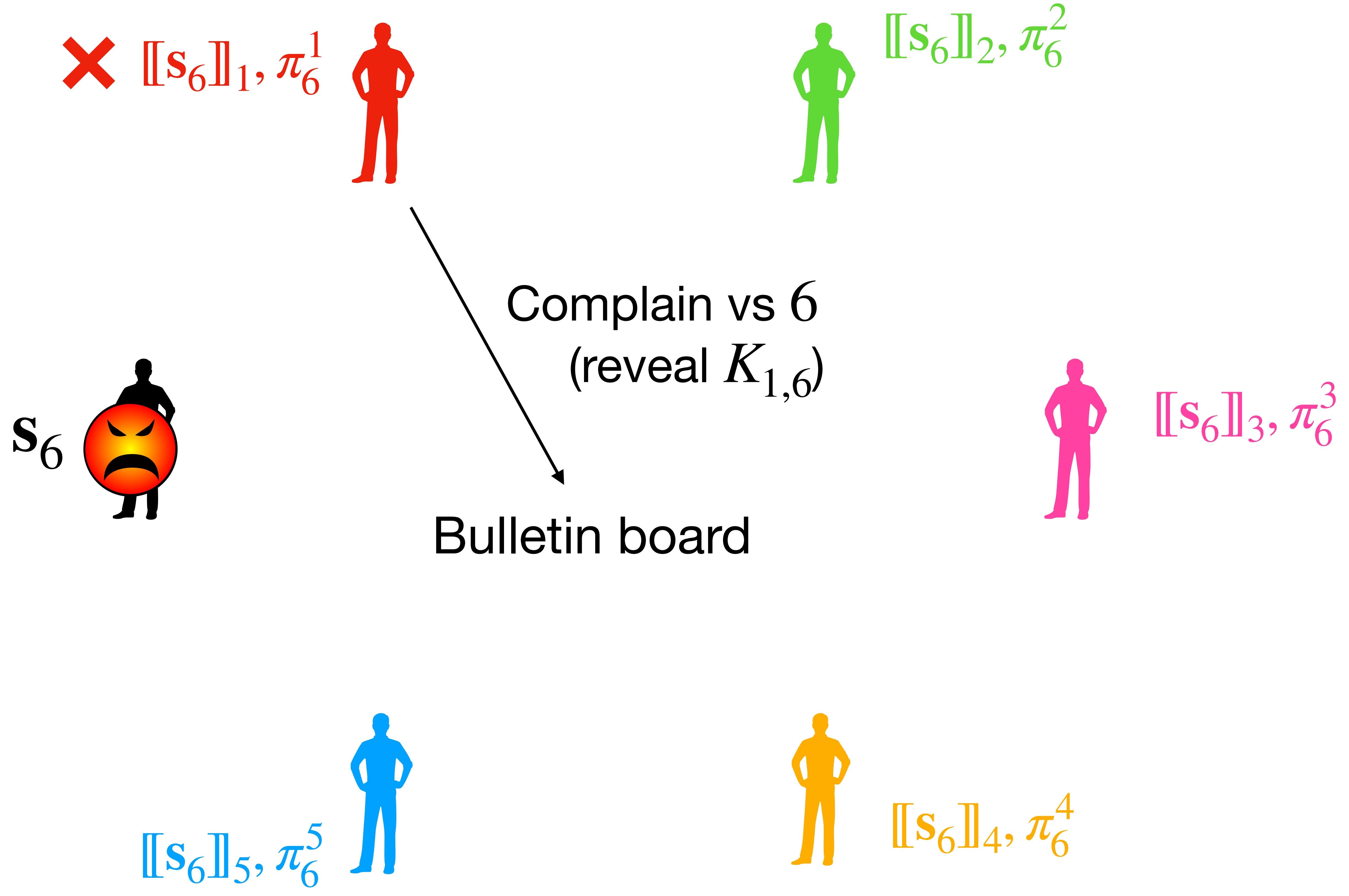
   1.c) Verify shares $([\![\mathbf{s}_i]\!]_j, \pi_i^j)_j$ and complain



Distributed Key Generation (DKG) from VSS

$$\mathrm{vk} = \mathbf{t} = \begin{bmatrix} \mathbf{A}' & \mathbf{I} \end{bmatrix} \cdot \mathbf{s} \in \mathcal{R}_q^k$$

where $\mathbf{A} = \begin{bmatrix} \mathbf{A}' & \mathbf{I} \end{bmatrix}$

$$\mathrm{sk} = \mathbf{s} \in \mathcal{R}_q^\ell \quad \text{short}$$

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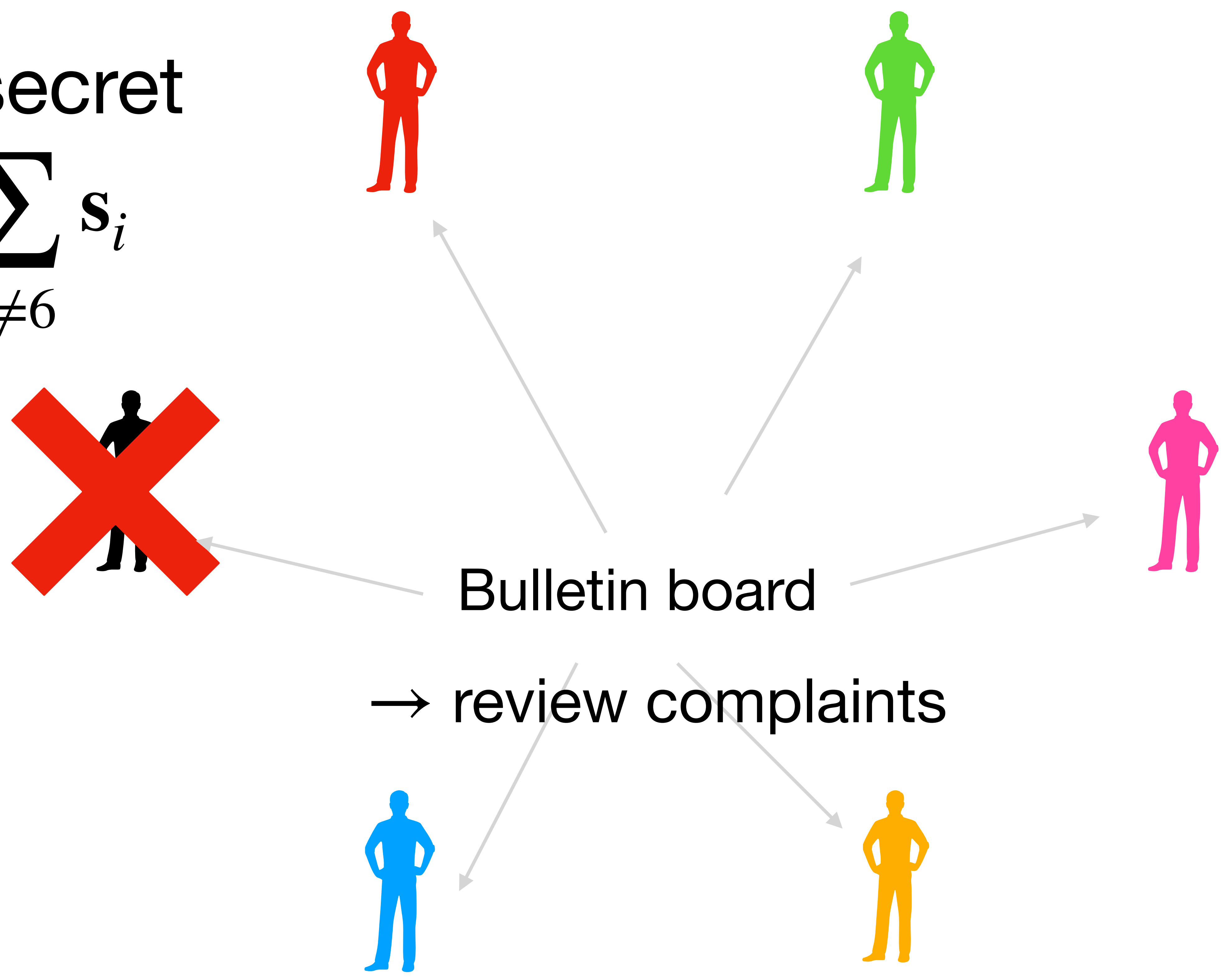
   1.c) Verify shares $([\![\mathbf{s}_i]\!]_j, \pi_i^j)_j$ and complain

   1.d) Aggregate

Final secret

$$\mathbf{s} = \sum_{i \neq 6} \mathbf{s}_i$$



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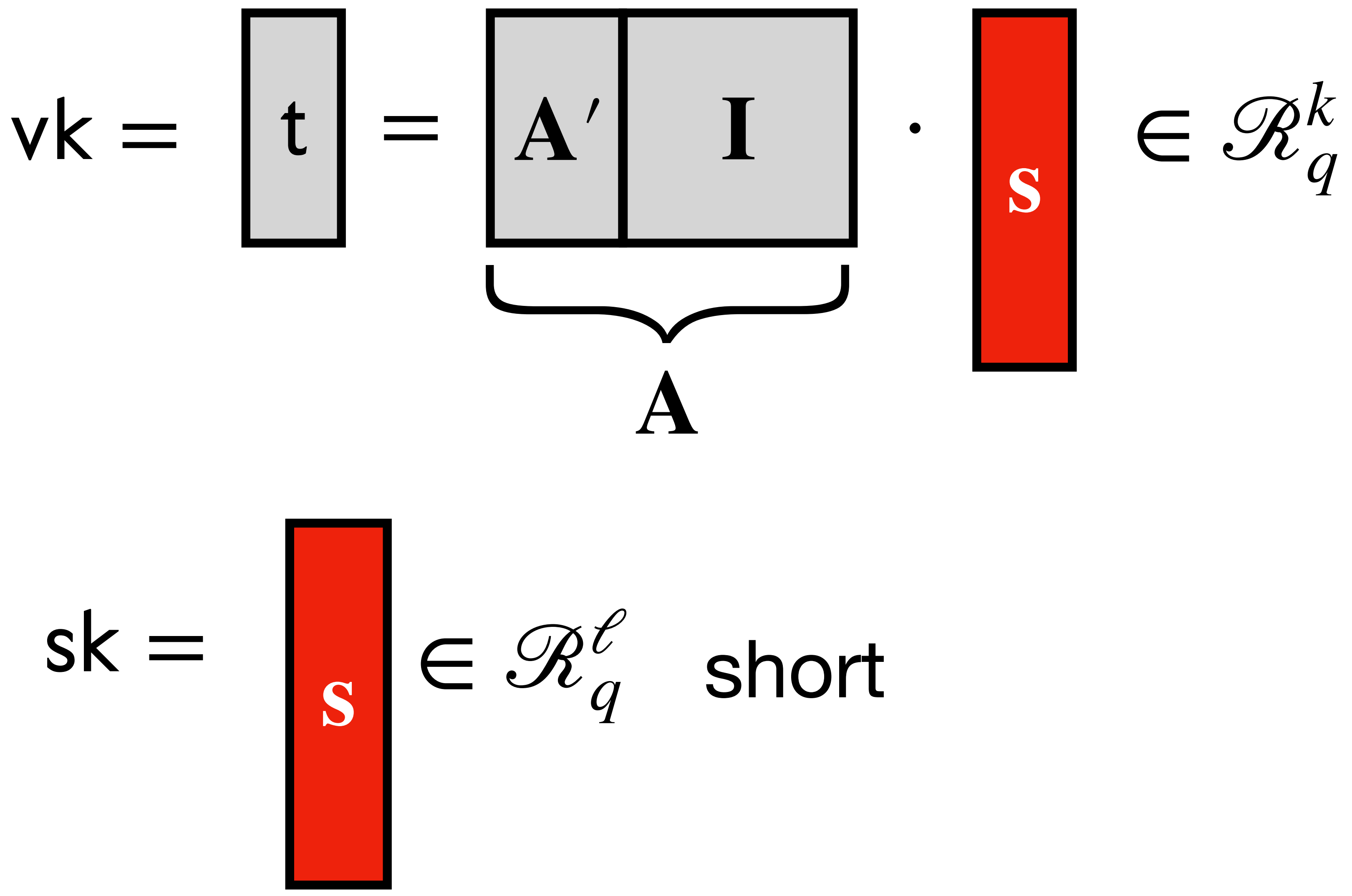

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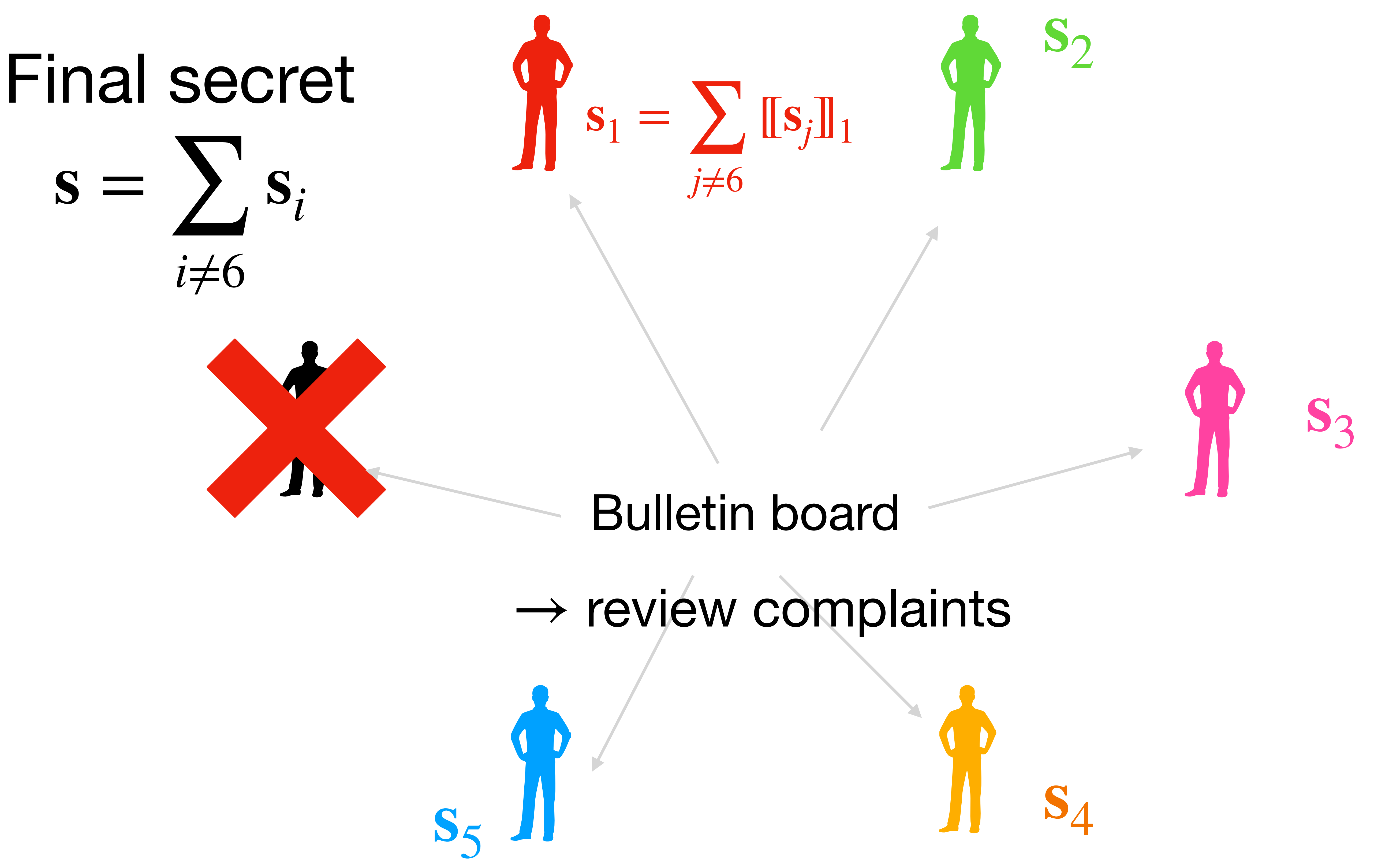

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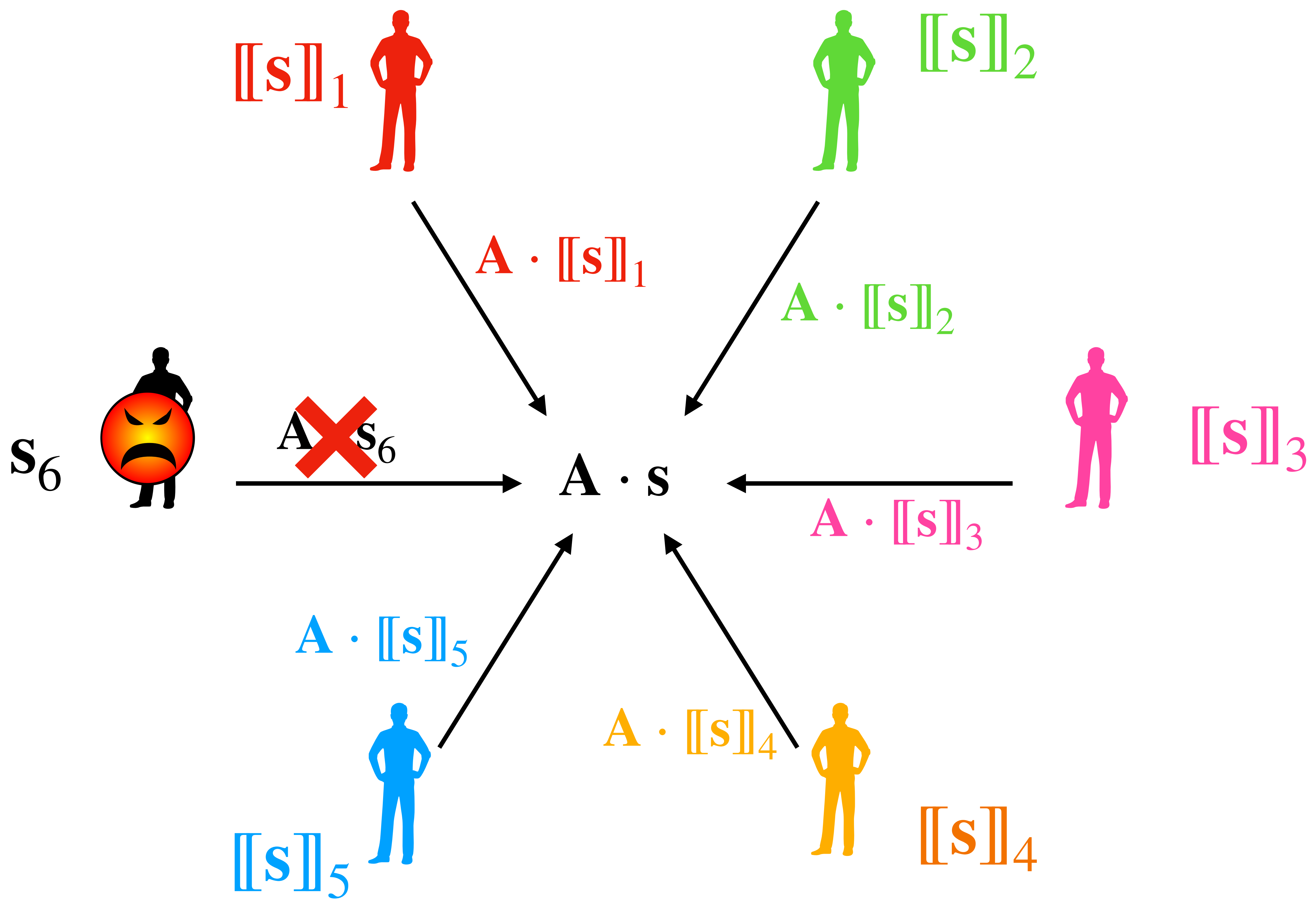

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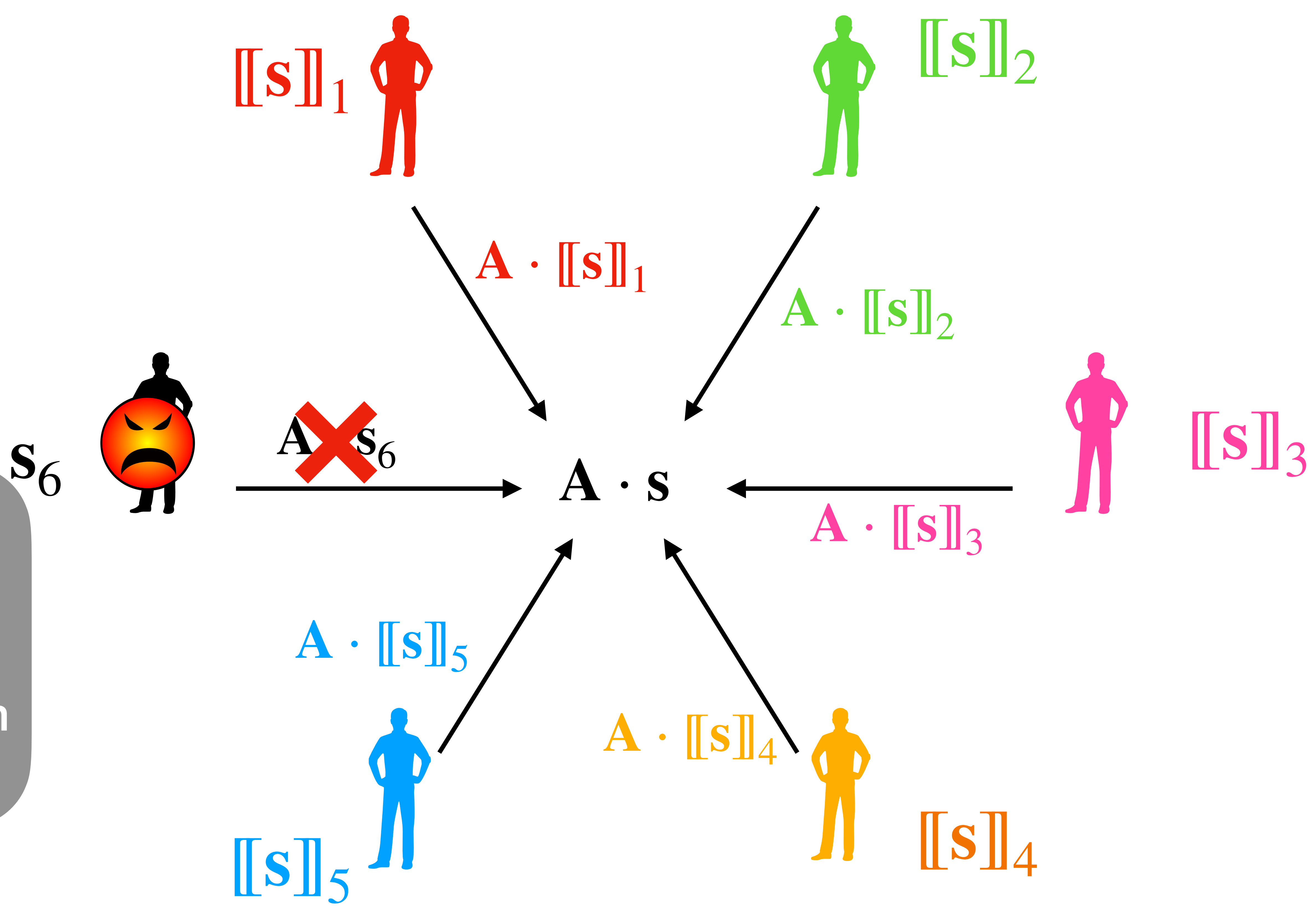
sk $= \boxed{\mathbf{s}} \in \mathcal{R}_q^\ell$ short

Use Reed-Solomon error correction to recover vk $= \mathbf{A} \cdot \mathbf{s}$

→ can only support T’=T/3 corruption



Robust Signing with VSS

Threshold Raccoon

$\mathbf{r_i} \leftarrow \chi$

$\mathbf{w}_i = \mathbf{A} \cdot \mathbf{r_i}$

$\mathsf{cmt_i} = H(\mathbf{w_i})$ →

← $(\mathsf{cmt}_j)_{j \in S}$

$\mathbf{w}_i$ →

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$$\mathbf{w} = \sum_{j \in S} \mathbf{w_j}$$

$$c = H(\mathsf{vk}, \mathsf{msg}, \mathbf{w})$$

$$[\![\mathbf{z}]\!]_i = c \cdot L_{S,i} \cdot [\![\mathbf{s}]\!]_i + \mathbf{r}_i + \Delta_i$$

$\mathbf{z}_i$ →

Robust Signing with VSS

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Robust ThRaccoon

1) Use DKG to sample secret $\mathbf{r} = \sum_i \mathbf{r}_i$ and compute $\mathbf{w} = \mathbf{A} \cdot \mathbf{r}$: 3 rounds

Robust Signing with VSS

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Robust ThRaccoon

1) Use DKG to sample secret $\mathbf{r} = \sum_i \mathbf{r}_i$ and compute $\mathbf{w} = \mathbf{A} \cdot \mathbf{r}$: 3 rounds

2) Compute signature shares: 1 round

$$c = H(\mathsf{vk}, \mathsf{msg}, \mathbf{w})$$

$$[\![\mathbf{z}]\!]_i = c \cdot [\![\mathbf{s}]\!]_i + [\![\mathbf{r}]\!]_i$$

If corruption threshold $T' \leq T/3$, *Reed-Solomon error correction* guarantees signature output.

3. A practical VSS with approximate shortness proof

Prior work on VSS

- Classical setting (uniform secret)
  - BGW VSS [BGW88]: IT security
  - Pedersen VSS [Ped92]: relies on DL
  - [ABCP23] based on hash functions
- VSS with shortness proof [GHL21]: quite large and DL aggregation

Our VSS

How to prove shortness of a vector $\mathbf{s}$ without revealing it?

Use a random projection to a smaller space!

Modular Johnson-Lindenstrauss lemma with offset [Ngu22]: Take a vector $\mathbf{y}$.

If a matrix $\mathbf{R}$ is sampled from a discrete distribution with coefficients ± 1 with proba $\frac{1}{4}$, and 0 with proba $\frac{1}{2}$.

Then, $\|\mathbf{R} \cdot \mathbf{s} + \mathbf{y} \bmod q\|_2$ is at least as large as $C \cdot \|\mathbf{s}\|_2$ for some $C = \omega(1)$.

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Use small Gaussian noise keeping enough entropy in $\mathbf{s}$ instead of information theoretic.

Our VSS

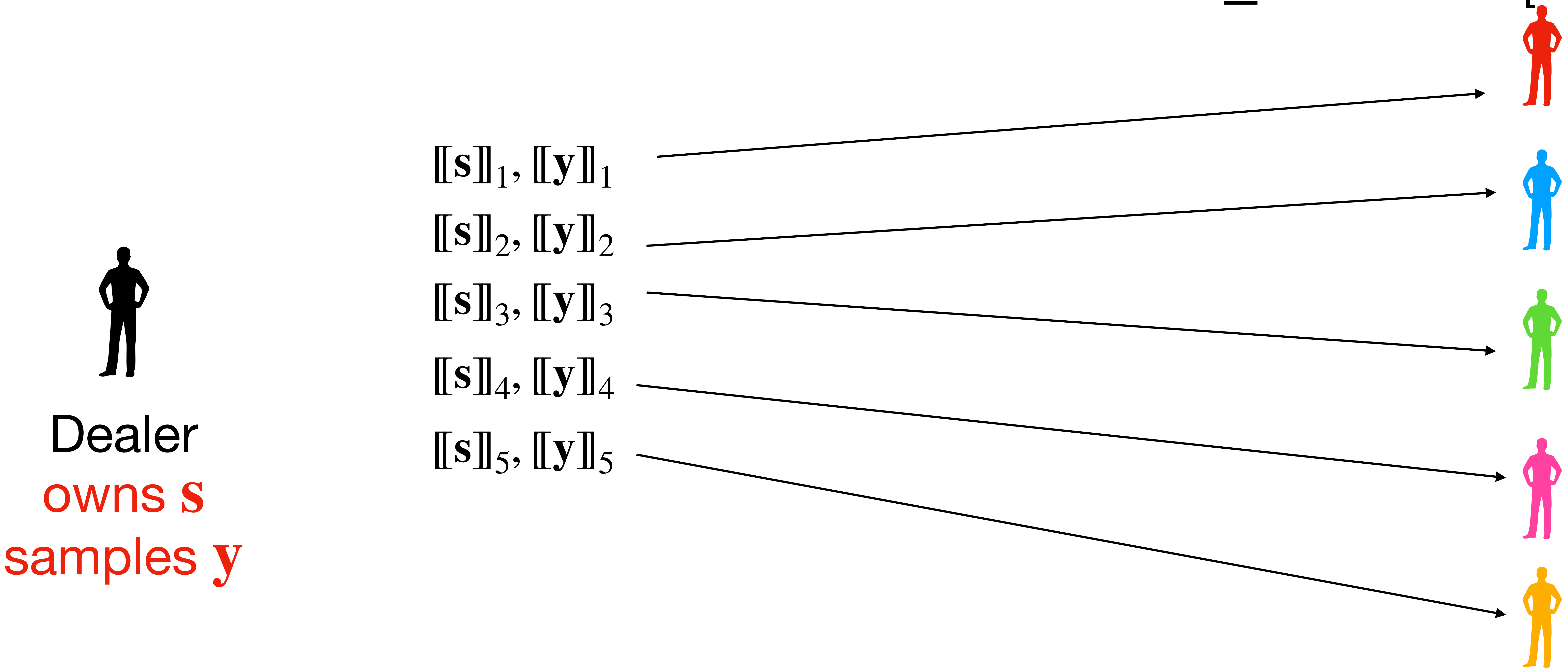
⚠ Johnson-Lindenstrauss only applies if $\mathbf{R}$ is sampled after $\mathbf{s}$ and $\mathbf{y}$.

→ Solution: hash-based verifiable randomness for $N \geq 2T'$ akin to [ABCP23].

Our VSS

Johnson-Lindenstrauss only applies if $\mathbf{R}$ is sampled after $\mathbf{s}$ and $\mathbf{y}$.

→ Solution: hash-based verifiable randomness for $N \geq 2T'$ akin to [ABCP23].



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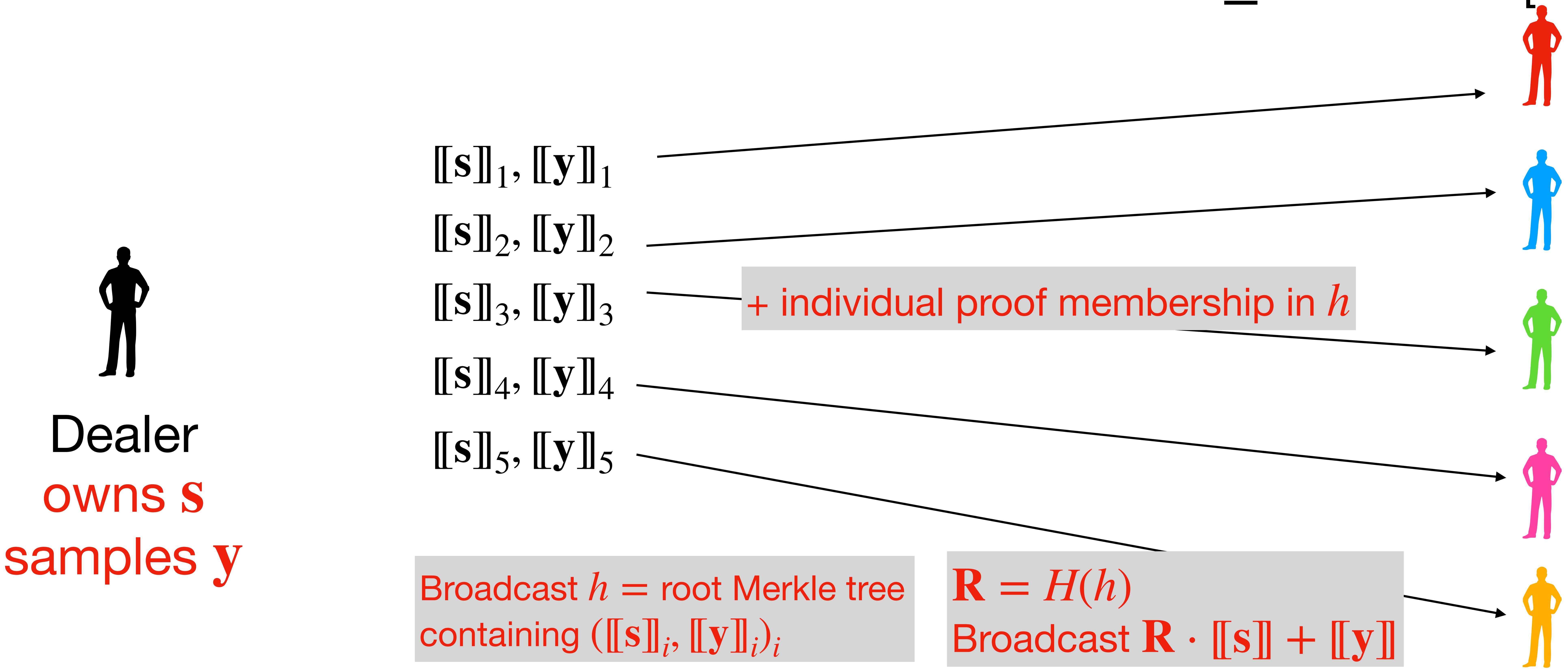

Our VSS

- Our VSS reveals $\mathbf{R} \cdot \mathbf{s} + \mathbf{y}$ where $\mathbf{y}$ is Gaussian: smaller shortness gap compared to rejection sampling.
  - Not purely ZK

Zero-knowledge:

$$\left.\begin{array}{l} \pi, ([\![x]\!]_i, \pi_i)_{i=1,\ldots,N} = \mathsf{VSS.Share}(\mathbf{x}) \\ \pi, ([\![x]\!]_i, \pi_i)_{i=1,\ldots,T-1} = \mathsf{SimShare}() \end{array}\right\} \quad \pi, ([\![x]\!]_i, \pi_i)_{i=1,\ldots,T-1} \quad \text{is indistinguishable}$$

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Zero-knowledge:

$\pi, ([\![x]\!]_i, \pi_i)_{i=1,\ldots,N} = \mathsf{VSS.Share}(\mathbf{x})$

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} $\pi, ([\![x]\!]_i, \pi_i)_{i=1,\ldots,T-1}$ is indistinguishable

Fragmentary knowledge:

$\pi, ([\![x]\!]_i, \pi_i)_{i=1,\ldots,N} = \mathsf{VSS.Share}(\mathbf{x})$

$\pi, ([\![x]\!]_i, \pi_i)_{i=1,\ldots,T-1} = \mathsf{SimShare}(\mathbf{R} \cdot \mathbf{x} + \mathbf{y})$

} $\pi, ([\![x]\!]_i, \pi_i)_{i=1,\ldots,T-1}$ is indistinguishable

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- **Approximation gap ~70**, vs $\gg 2500$ in [GHL21] using JL lemma

4. Bonus: application to hash-and-sign

Fiat-Shamir vs Hash-and-Sign signatures

Fiat-Shamir

… Dilithium, Raccoon

$\mathbf{r} \leftarrow \chi$

$\mathbf{w} = \mathbf{A} \cdot \mathbf{r} \xrightarrow{\mathbf{w}}$

$\longleftarrow \quad c = H(\mathsf{vk}, \mathsf{msg}, \mathbf{w})$

$\mathbf{z} = c \cdot \mathbf{s} + \mathbf{r} \longrightarrow$

Accept if

- $\mathbf{z}$ is short
- $\mathbf{A} \cdot \mathbf{z} = c \cdot \mathbf{t} + \mathbf{w}$

Hash-and-Sign

… Falcon, Plover

$\mathbf{u} = H(\mathsf{vk}, \mathsf{msg})$

$\mathbf{z} = \mathsf{Inv}(\mathsf{sk}, \mathbf{u})$

Accept if

- $\mathbf{z}$ is short
- $\mathbf{A} \cdot \mathbf{z} = \mathbf{u} \quad (= H(\mathsf{vk}, \mathsf{msg}))$

Plover signature scheme

Based on Eagle [YJW23]

$$\mathsf{vk} = \mathbf{t} = \underbrace{\begin{bmatrix} \mathbf{A}' & \mathbf{I} \end{bmatrix}}_{\mathbf{A}} \cdot \mathbf{s} - 2^{\nu} \in \mathscr{R}_q^k \qquad \mathsf{sk} = \mathbf{s} \in \mathscr{R}_q^{\ell} \text{ short}$$

$$\mathbf{u} = H(\mathsf{vk}, \mathsf{msg})$$

$$\mathbf{r} \leftarrow \chi$$

$$\mathbf{w} = \mathbf{A} \cdot \mathbf{r}$$

$$\mathbf{u}' = \mathbf{u} - \mathbf{w} = 2^{\nu} \cdot c_1 + c_2$$

$$\mathbf{z} = \begin{bmatrix} c_1 \cdot \mathbf{s} + \mathbf{r} \\ c_1 \end{bmatrix}$$

Accept if

- $\mathbf{z}$ is short
- $[\mathbf{A} \quad \mathbf{t}] \cdot \mathbf{z} = \mathbf{u} \quad (= H(\mathsf{vk}, \mathsf{msg}))$

Conclusion

Conclusion

- Framework relying on VSS to achieve robust DKG and robust signature scheme with corruption threshold $T' = T/3$.
- **Pelican:** first lattice hash-and-sign threshold signature + DKG + robustness

Pelican = application to Plover, in this presentation applied to Raccoon

- Practical VSS scheme with approximate shortness proof: slack ~70

κ	**max T'**	**| vk |**	**| sig |**	**Communication**
128	16	12.8kB	12.3kB	26.8 + 19N kB
196	1024	25.6kB	26.4kB	53.8 + 38N kB

Proposed parameter sets for Pelican